# The Millikan oil drop experiment

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## 1 Abstract

In the Millikan oil drop experiment the magnitude of the elementary charge was measured to be $(1.70 \pm .09) \cdot 10^{-19}$ C. The tabulated value is $1.6022 \cdot 10^{-19}$ C. The elementary charge was found by examining the motion of oil droplets with radii $\sim 10^4$ cm between a pair of highly charged capacitor plates. Although the droplets typically contained $\sim 10^{12}$ protons, they were found to have net charges of only 2 to 15 times the elementary charge. Such low charges enabled observation of the quantization of charge.

## 2 Background

The Greek philosophers debated whether matter is infinitely divisible, or composed of discrete atoms (the word atom means "indivisible"). In the 19th century the atomic idea was the basis for several successful theories of chemistry and physics, such as Dalton's theory of chemical combination and Maxwell's kinetic theory of gases. Further Einstein's theory about quantization of energy and Rutherford's model of the atom inspired Millikan to examine the possible quantization of electric charge through this experiment.

## 3 Theory

When a particle is falling freely, it is acted upon by the gravitational force minus the force of buoyancy:

$$F_{gb} = \frac{4}{3}\pi r^3(d - da)g \tag{1}$$

These forces balance the force of viscosity giving:

$$\frac{4}{3}\pi r^3(d - da)g = 6\pi r \mu u_d(1 - a/r) \tag{2}$$

Defining r' to be:

$$r' = \sqrt{9\mu u_d/2(d - d_a)g} \tag{3}$$

yields a droplet radius:

$$r = r'\sqrt{(1 - a/r')} \tag{4}$$

Equations 3 and 4 show that knowing the velocity of free fall and the parameters associated with the small size of the droplets, the radius of the droplet can be calculated directly.

When the electric field is turned on, an equilibrium between electric and other forces is soon reached, which makes the droplets move with a constant velocity. If the direction and magnitude of the electric field is appropriate, the droplets will move slowly upwards. In that case, the following equation is obtained:

$$\frac{4}{3}\pi r_3(d - d_a)g - \frac{Vne}{s} + 6\pi\mu u_r(1 - a/r) = 0 \tag{5}$$

where V is the potential of the capacitors, s the separation, ne the charge of the oil droplets and $u_r$ the rise velocity of the droplets.

By combining 3 and 5 the following equation is found:

$$ne = \frac{6\pi\mu r' s}{V}(u_d + u_r)(1 - a/r')^{3/2} \tag{6}$$

Defining e' as:

$$e' = \frac{6\pi\mu r' s}{n}(u_d + u_r) \tag{7}$$

gives that :

$$e'^{2/3} = e^{2/3}(1 + a/r') \tag{8}$$

By making repeated measurements of $e'^{2/3}$ as a function of $1/r'$ a graph is obtained from which the value of $e'^{2/3}$ as $1/r' \to 0$ can be extrapolated. Since $e' \to e$ as $1/r' \to 0$ the extrapolation will therefore yield the elementary charge.

Two problems arise when completing this procedure. First n has to be determined correctly when e' is determined. Secondly, it is necessary to extrapolate the graph of $e'^{2/3}$ as a function of $1/r'$ with a high level of accuracy. Both these problems require that a large number of observations be made. The first requires a large number so that a clustering around certain values is found, and the second so that we have observations for a broad band of $1/r'$.

# 4 Apparatus and procedure

The main body of the experimental apparatus were a pair of capacitor plates. They were separated by three "spacer balls", whose diameter determined the separation of the capacitor plates. The capacitors were kept in a chamber, so that no external source affected the motion of the droplets. A light source made it possible to look at the motion of the droplets through a microscope which was placed in the chamber. In the lens of the microscope there were two lines, whose separation was measured on beforehand. The velocity of the droplets could then be found by timing the droplets as they traveled the predetermined distance between the lines.

Through a small hole in the upper capacitor oil droplets were sprayed in between the plates. Oil droplets with varying velocities (and hence radii) soon became visible in the microscope. The practical part of the experiment then consisted of "trapping" droplets of suitable radius and charge between the two lines of the microscope. A mechanism for turning the field on and off quickly made this possible. Individual droplets were then followed as they made several complete cycles up and down. Timing the motion, and recording the potential (which was typically around 3000V), gave a sufficient number of parameters to determine r' and e'.

# 5 Observations and Analysis

Tables including the measured rise and fall times can be found in the end of the paper. The table also contains the calculated values for r' and q' based on the measurements and the formulae which can be found in the theory section.

## 5.1 Qualitative observation of quantization of charge

Already while performing the measurements, qualitative indications of the quantization of charge were found. The droplets were followed as they made many complete cycles between the capacitor plates. A few times while a droplet was moving upwards, the droplet suddenly changed its speed and the speed was established at a new level. Since such phenomena only took place when the droplets were moving upwards, it seems reasonable to assume that they were caused by a change in the charge of the particle. Since the change in the velocity always was of a distinct magnitude, the changes in the charge must have had a finite magnitude, which indicated that the charges involved in the experiment were quantized. If the charge had not been quantized, we would rather expect that the charge would 'leak out' slowly, and thereby decrease the velocity gradually.

## 5.2 Quantitative analysis

Figure 1 shows $q^{2/3}$ as a function of $1/r'$. The plot clearly indicates the quantization of charge, because only certain 'levels' in the values of $q'^{2/3}$ are seen. The problem for the further analysis is assigning a specific number of elementary charges to each of the lines (determining n in equation 8), so that $e^{2/3}$ can be determined from $q^{2/3}$. Assigning the quantum numbers is the essence of he whole experiment, because that directly yields the quantization of charge.

A naive approach to this step would be to assign n=1 to the lowest level. However, the magnitude of the elementary charge may very well be so small

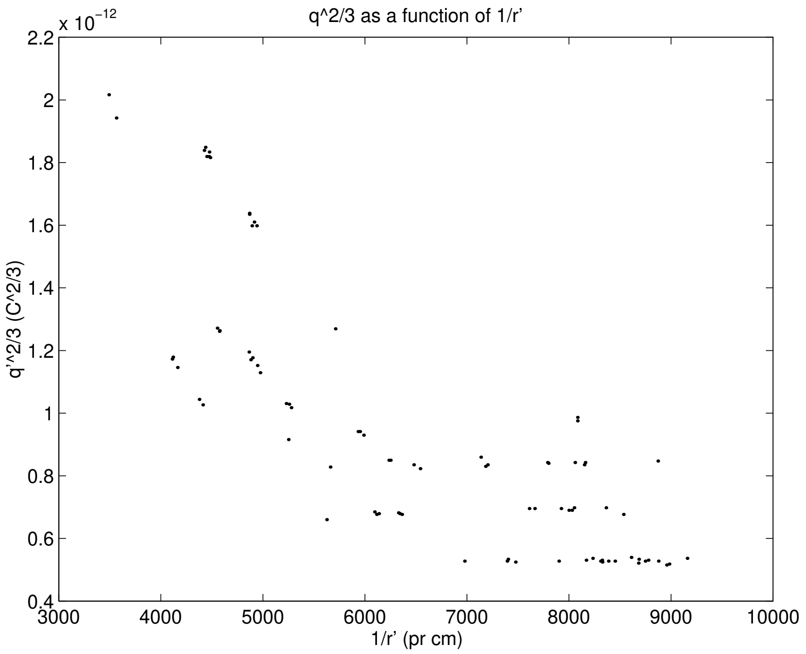


Figure 1: Quantized levels of charge

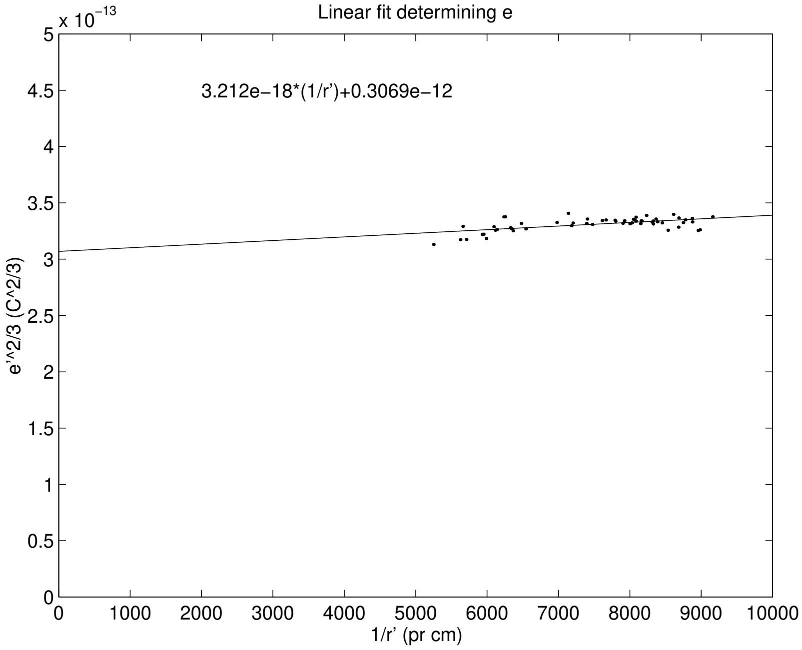


Figure 2: Linear fit yielding the elementary charge

that the electric field of the capacitor plates would be too weak to overcome the gravitational pull on droplets of the size involved in this experiment. A better approach is to estimate an average $q'^{2/3}$ of the lowest line, and also to look at the average spacing between the proceeding lines. Since $q^{2/3}$ is approximately twice as big as the spacing between the consecutive lines, it is fair to assume that q' on the lowest level corresponds to 2e'. Thus the quantum number 3,4..... can then be assigned to the consecutive lines. By dividing through with the determined quantum numbers, the graph in figure 2 is obtained. In the graph, lines with $n \geq 6$ have been omitted, because uncertainties don't make the lines as distinct as they ideally should be. Thus, even though the lines are not clearly separated, they would, if they were included, contribute to determining the elementary charge without showing any indication of such an elementary charge themselves.

By fitting a line to the points in figure 2 and extrapolating it to $r' \rightarrow \infty$, the value $3.186 \cdot 10^{-13}$ is obtained. The main sources of error associated with this number are estimated to be: 2% uncertainty in the voltage across the capacitor plates, 1% uncertainty in the viscosity due to variations in the temperature, 2% uncertainty in the timing of the motion and 2% uncertainty due to the fit itself. Since all these parameters enter the formula for $e^{2/3}$ (equation 7) in the first order, the total uncertainty is found by taking:

$$\sigma = \sqrt{\sigma_V^2 + \sigma_{temp}^2 + \sigma_{fit}^2 + \sigma_{time}^2} \approx 3.6 \tag{9}$$

To get the value for e the value obtained by the fit must be raised to the 3/2 power. This also *multiplies* the error with a factor 1.5, giving a total error of $\sim 5\%$ The following value for the elementary charge is thus obtained:

$$e = (1.70 \pm 0.09) \cdot 10^{-19} C \tag{10}$$

The experimental value is in agreement with the tabulated value, $e = 1.6022 \cdot 10^{-19} C$.

# 6 Conclusions

Millikan's oil drop experiment has showed that the electric charge in oil drops is quantized, and has allowed a determination of its value to $e = (1.70 \pm 0.09) \cdot 10^{-19} C$. The experiment has neither proved that the value of the fundamental electric unit is on the order of $e = 1.70 \cdot 10^{-19} C$ nor that electric charge in itself is quantized. All it has proved is that electric charge *appears* in quanta of magnitude $e = 1.70 \cdot 10^{-19} C$ in oil droplets. Other experiments which followed Millikan's experiment, like Johnson's shot noise experiment, indicated the same quantization of charge as the one found in this experiment. One might therefore believe that the discussion on whether the charge is quantized in quanta of $1.6022 \cdot 10^{-19} C$ or appears in quanta of $1.6022 \cdot 10^{-19} C$ is a purely philosophical one, with little relevance to physics. However, with the emergence of elementary particle physics, it has been showed that there do exist particles with a lower charge than the 'elementary' charge. These particles have a charge of 1/3 times the elementary charge, but under 'normal' conditions they always appear in groups of three, so the elementary charge unit in 'everyday physics' is indeed the one observed above.