Massachusetts Institute of Technology

Department of Physics

Physics 8.04 October 12, 1995

Quantum Physics I, Spring 1995

As in the syllabus, "FT"=*French and Taylor*, "ER"=*Eisberg and Resnick*, "FLP"=*Feynman Lectures, vol. III*, and the numbers refer to sections. Here the numbers listed are the pre-lecture readings. The "extended" readings are also given, but within parenthesis. Reading assignments:

Tue. Feb. 21: No lecture. (Institute follows Monday schedule.)
Thurs. Feb. 23, Lecture 5: FLP 1.1-1.8 (FT 2.10-2.11, FLP 2.1).

Problem Set II

(Due Friday, February 24 at 5:00 pm sharp.)

Work requested of the students is highlighted by either a question mark or underlining.

1 (7 pts) "True" Rutherford Scattering

In the notation of problem 1.5 from the first problem set, classical analysis gives the differential scattering cross section for a repulsive $\frac{zZe^2}{r}$ Coulomb potential to be

$$\frac{d\sigma}{d\Omega} = \frac{(D/4)^2}{\sin^4(\theta/2)} \tag{1}$$

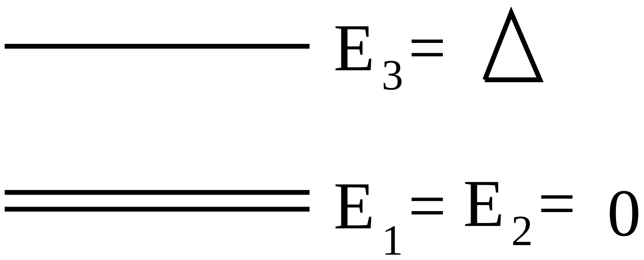
where $D \equiv \frac{zZe^2}{Mv^2/2}$ is the distance of closest possible approach of an alpha particle of charge $ze = 2e$, mass M and traveling at velocity v to a heavy nucleus of charge Ze. Note that, unlike the case of scattering from hard spheres, the cross section depends on both the velocity of the incoming particles and the scattering angle. Using the same approach as in the solutions to problem 1.5, answer the following two questions:

a) A beam of α-particles, of kinetic energy 5 MeV and intensity 10^5 particle/sec, is incident normally on a gold foil of thickness 2.0×10^{-5} cm. (The density, atomic weight and atomic number of gold are 19.3 g/cm^3, 197 and 79 respectively.) An α particle counter of area 0.25 cm^2 is placed on the opposite side of the foil from the incoming beam at a distance of 100 cm at an angle of 30^o from the incoming flux. How many counts should we expect per hour if the detector registers all particles passing through it? How many counts will there be per hour for back-scattering events 150^o away from the direction of incoming flux?

b) The density and atomic weight of copper are 8.9 g/cm^3 and 63.6, respectively. If Rutherford scattering through a copper foil of thickness 2.0×10^{-5} cm at $\theta = 10^o$ using the same detector and α-particle source as in (a) gives 46 counts per hour, what then do you deduce for the charge on the copper nucleus?

2 (7 pts) Computing Thermal Averages

Consider a three-state system as indicated in the diagram in Figure 2. Here the first two states, 1 and 2, have the same energy. (This situation is referred to as a "degeneracy".)



a) What do you expect the average energy of this system to be at absolute zero temperature ($T = 0$)? At extremely high temperatures ($T \to \infty$)? Why?

b) To confirm your expectations, apply the Boltzmann distribution to <u>write</u> the probability of each of the three states as $P_i = \alpha e^{-\beta E_i}$, where α is the proportionality constant and $\beta \equiv \frac{1}{kT}$, where k is Boltzmann's constant. Use the condition that these probabilities must sum to unity (the normalization of the probability distribution) to <u>determine</u> α. Finally, <u>compute</u> the average energy of this system at temperature T and <u>verify</u> your predictions in a) for the behavior of this system as $T \to 0$ and $T \to \infty$.

c) <u>Confirm</u> that the shortcut of summing the *partition function*, $Z \equiv \sum_n e^{-\beta E_n}$, and then taking

$$< E >= -\frac{\partial}{\partial \beta} \ln Z$$

gives the correct result.

3 (10 pts) Thermal Radiation and the Photoelectric Effect

a) Consider a 100 $Watt$ ($10^9 erg/s$) light bulb with a Tungsten (melting temperature $3683^o K$) filament. Assuming that the bulb is designed to operate at $3500^o K$, and that nearly all of the 100 $Watt$ of electrical power consumed by the bulb is converted to thermal radiation. What then must be the surface area A of the filament in cm^2? (Hint: Stefan's constant is $\sigma = 5.67 \times 10^{-5} erg/cm^2/s\text{-}^o K^4$.) <u>Explain</u> whether your result for A is reasonable from what you would expect the area of a filament to be.

b) What fraction of the thermal radiation from the bulb falls in the visible part of the spectrum ($\lambda = 430nm \to 690nm$)? Do you consider the incandescent bulb to be an energy efficient light source?

Hints: Change variables to express the integral in the form $\int dx \, \frac{x^3}{\exp(x)-1}$, and then make (and <u>justify</u> by giving an error estimate) the approximation $\frac{x^3}{\exp(x)-1} \sim x^3 e^{-x}$. Also, as mentioned in the lecture notes on Black Body radiation,

$$\int_0^\infty \frac{x^3 \, dx}{e^x - 1} = \frac{\pi^4}{15}, \tag{2}$$

as may be derived by a power series expansion of the denominator. Finally, you may wish to have the values

$$\begin{aligned} h &= 6.63 \times 10^{-27} erg \; s \\ k &= 1.38 \times 10^{-16} erg/^o K \end{aligned}$$

c) The same bulb is then used as the light source in a photoelectric effect apparatus with a metal target with work function $5.6eV$ and surface area $a = 1cm^2$ placed $D = 10cm$ away from, and directly facing, the bulb. How many photons per second, $N_\gamma(\nu)d\nu$ with frequencies in the interval $(\nu, \nu + d\nu)$ hit the target? Assuming that all photons in this problem that are capable of ejecting an electron from the surface do so and that all of these electrons are eventually collected by the electrode, what will be the maximum saturation current (in units of $1\ nano - Ampere \equiv 6.25 \times 10^9 electron/s$) observed in the experiment? (You may use the same type of approximation you developed in part (b)).

4 (10 pts) Measuring Planck's Constant Sitting at Your Desk

In this problem, you will explore the plausibility of the responsibility of photon shot noise for the "night-time reflection effect" and then use that effect to make an arm-chair estimate of the value of Planck's constant h.

We will use the following assumptions about the human eye:

1. The "response/sampling" time τ for the human eye is $0.02s$, somewhere between the imperceptible cycle time of fluorescent lighting and the noticeable flicker of movies at 15 frames per second.
2. The resolution of the human eye is $d\theta = 1 arc\,min \equiv \left(\frac{1}{60}\right)^o$ (the angle of a single pixel on a 1024×1024 CRT at about $100cm$), corresponding to a solid angle of $\Omega = d\theta^2$.
3. The radius of the pupil (the aperture through which light enters the eye) is $0.4cm$

In (a)-(c) it is convenient to express your answers in terms of numerical constants and an unknown parameter Δ, the energy/photon expressed in units of $1eV = 1.60 \times 10^{-12} erg$.

a) If we take the average intensity of indoor lighting to be that of the uniform illumination of the 100 $Watt$ light-bulb of problem **3**, at a distance of 3 meters, how many photons/s in the visible region of the spectrum illuminate each cm^2 in the room. (For simplicity, use your result from the previous problem for the total energy flux and make the approximation that all the photons in the visible range carry the same energy, which we will take for now to be $(1eV) \times \Delta$. We expect but do not yet assume that Δ is on the order of unity.) In terms of Δ, assuming that objects reflect the light impinging on them uniformly in all directions, how many photons/s are received by each receptor in the eye? (See Figure 4 as an aid in your thinking.) Again, in terms of Δ, how many photons are impinging during a single response time, $N_\tau(indoor)$?

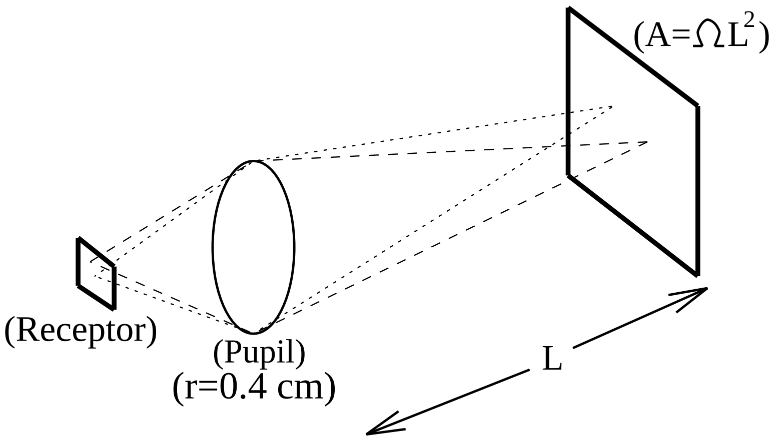
b) Now, suppose that ten percent (0.1) of the light-energy impinging on a glass window is reflected back. How many photons (on average) per sampling time are received by each receptor from the reflection, $N_\tau(refl)$, if the room is illuminated under the conditions (and approximations) described above? What size shot noise fluctuations, $\sigma_N(refl)$, do you expect about this average?

c) If we take the average outdoors night-time lighting condition to correspond to the illumination of the same $100 Watt$ bulb but at an average distance of 30 meters, what number of photons is expected to arrive at each receptor in a sampling time from objects outside the window, $N_\tau(outdoors)$?

d) Suppose that under these conditions, you had observed that you could just barely make out images from the outside, what relationship must exist between your final answers to (b) and (c)? Using this relationship, determine the value of the unknown parameter Δ and give absolute numbers for your responses to (b) and (c). Using the fact that the wavelength of visible light is on the order of 5100Å, what order of magnitude would you then assign to Planck's constant h? How close is this to the accepted value?

5 (8 pts) Time Delay in the Photoelectric Effect

Do problem 1.9 from *French and Taylor*.



6 (5 pts) Photon momentum

At what rate would a hydrogen atom have to absorb (visible) photons of energy $h\nu = 2.55eV$ directed at it from below in order to suspend it against the earth's gravity? (Assume that after absorbing each photon, the atom immediately emits it in a *random* direction.)