Massachusetts Institute of Technology

Department of Physics

Physics 8.04 October 12, 1995

Quantum Physics I, Spring 1995

As in the syllabus, "FT"=*French and Taylor*, "ER"=*Eisberg and Resnick*, "FLP"=*Feynman Lectures, vol. III*, and the numbers refer to sections. Here the numbers listed are the pre-lecture readings. The "extended" readings are also given, but within parenthesis. Reading assignments:

Tue. Feb. 28, Lecture 6: FLP 1-8, 2-2, 2-4 (FLP: 2-3, 2-6).
Thurs. March 2, Lecture 7: FT 1-7, 1-9, 1-12 (FT: 1-8, 1-10).

Problem Set III

(Due Friday, March 3 at 5:00 pm sharp.)

Work requested of the students is highlighted by either a question mark or underlining.

1 (5 pts) Typical de Broglie wavelengths

Do problem 2-6 from *French and Taylor*.

2 (10 pts) de Broglie waves in a potential

In lecture, we considered the correspondence between particles moving in free space according to an energy function $E(p)$ and wave-packets propagating under a dispersion relation given by $\nu = E$, $p = \frac{h}{\lambda}$. In this problem you will consider the same correspondence but for a particle in a system with potential energy.

Consider a particle whose energy is given by

$$H = \frac{p^2}{2m} + V(x) \tag{1}$$

Written this way, as a function of the momentum p, the energy function of the particle has a special name. It is known as the "Hamiltonian," and so we call the energy here "H" instead of the more familiar "E".

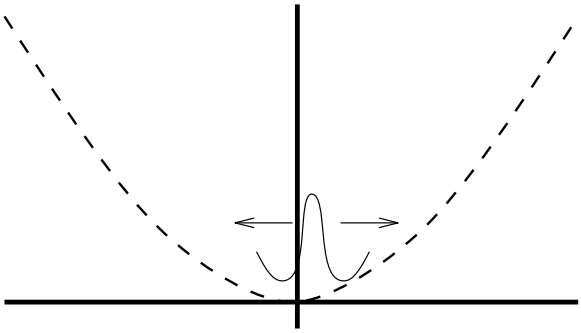
Imagine that an external force $F^{ext}(t)$ is applied to this particle in this system in such a way that the momentum of the particle as a function of time is given by $p(t)$.

a) Use conservation of momentum to determine $\frac{dp}{dt}(t)$ in terms of the force acting on the particle $F^{ext}(t)$, the position of the particle $x(t)$ and the potential function $V(x)$.

b) Use conservation of energy to balance the change in energy H of the particle against the rate at which the external force does work $F^{ext}v$, where $v(t) \equiv \dot{x}(t)$. Show that

$$\dot{x} = \frac{\partial H}{\partial p}. \tag{2}$$

Is this relation true even when the external force is zero?



c) Show that when the external force is zero we also have,

$$\dot{p} = -\frac{\partial H}{\partial x}. \tag{3}$$

Eq. (2) and Eq. (3) taken together are known as the canonical equations of Hamilton, and are studied in detail in 8.06.

d) Now consider a wave-packet description of the same particle. So long as the wavelength of the waves are much smaller than the scales over which $V(x)$ varies, we can imagine a very localized wave-packet in place of our particle. Around each location in space x where the packet can be centered, there is then a different *local* dispersion relation $\nu = f(\frac{1}{\lambda}, x)$. Given f, what velocity we would associate with a wave-packet centered about wavelength λ_o and position x_o?

e) For motion of the particle in the wave-packet description to be consistent with the motion determined in the particle description (Eq.'s (2-3)), what must be the form for $f(\frac{1}{\lambda}, x)$ in terms of $V(x)$ and λ? Show that we have again the same correspondence as in class, $E \leftrightarrow h\nu$ and $p \leftrightarrow h/\lambda$.

f) What local dispersion relation would you need to make a wave-packet oscillate back and forth in simple harmonic motion? (See Figure 2.)

3 (12 pts) Refraction of de Broglie waves

Do problem 2-7 from *French and Taylor*.

4 (7 pts) The Uncertainty Principle: Spreading of Wave-packets

Consider a particle of mass m that at time $t = 0$ is at $x = 0$ and traveling along the x-axis with a velocity v. It is described by a wave-packet of group velocity v and width w.

a) From the uncertainty principle, estimate the uncertainty in the *velocity* of the electron.

b) From the uncertainty in the velocity, estimate the uncertainty in the position of electron, $\Delta x = w(t)$, as a function of time $t > 0$ as the wave-packet then begins to move.

5 (7 pts) The Uncertainty Principle: The energy of confinement

To make color displays with semiconductor devices, workers try to increase the energy of electrons several volts by confining the electrons in very small crystals.

The behavior of an electron moving through a crystal is much like that of an electron moving through free space, *except* that because of the interaction between the electron and the crystal the electron acts as though it had a different mass, an "effective mass". In Germanium, for instance, this mass is $m_{eff} = 0.12m_e$, where $m_e = 9.11^{-28}g$ is the mass of an electron in free space.

Check the feasibility of this approach in Germanium by using the uncertainty principle to estimate the kinetic energy of an electron of mass m_{eff} confined to a cube of side L. How does the energy of the electron vary as L decreases? What size L is needed to increase the energy of the electron by $1eV$? Is this reasonable or is it smaller than the average spacing between atoms in Germanium? (The density of Germanium is 5.32 g/cm^3 and its atomic weight is 72.6 $g/mole$.)

6 (7 pts) Computer Assignment # 2: Two-slit experiments

In this problem you will investigate quantum wave phenomena using an interactive program on Athena. To run this program, log into your athena account on an Athena Sun Station (preferably a SPARCstation 5), and execute:

athena% **add 8.04**
athena% **cd PS3**
athena% **prob6**

The program will give you instructions from there.

We recommend that you try the new SPARCstation 5 cluster in 2-032, which contains fifteen new color machines. If you have any difficulties whatsoever, do not hesitate to ask for assistance, writing email to muchomas@athena.mit.edu, calling Prof. Arias at x3-6831, or dropping by his office, 12-110 at any time.