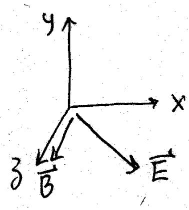
P214

Solution to Prelim 2 11/8/01

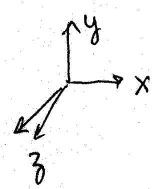
(1) (a)



(b) $|\vec{E}| = \sqrt{2}E_0$

$B = \sqrt{2}E_0/c$

(c)



Direction $= -\hat{i} - \hat{j} = -\hat{x} - \hat{y}$

in xy plane

(d) not enough information $\boxed{C}$

(2) (a) y-direction (b) after 45° grid

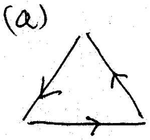
$I_0 \longrightarrow I_0/2 \quad (I_0 \cos^2\theta)$

$\hookrightarrow I_0/4$ $\boxed{D}$

vertical grid

(c) $I_0 \longrightarrow 0 \longrightarrow 0$ $\boxed{B}$

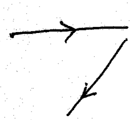
(3) (a)



yields $I_{min} = 0$

(b) $I_{min} = I_0 \left| 1 + e^{i 2\pi/3} + e^{i 4\pi/3} \right|^2$

(c)

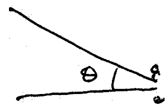


gives $I = I_0$

(4) Use phasor diagram for $I_1 \oplus I_2$

$\left.\begin{array}{l} \underset{\sqrt{I_0}}{\longrightarrow}\underset{3\sqrt{I_0}}{\longrightarrow} = \underset{4\sqrt{I_0}}{\longrightarrow} \\ \longleftarrow\!\!\!\!\longrightarrow \; = 2\sqrt{I_0} \end{array}\right\}$ $16I_0/4I_0$

$\boxed{F}$

(5) (a) 

$$k\Delta R = 2\pi = k d \sin\theta$$

$$d = \frac{2\pi}{k \sin\theta} = \frac{\lambda}{\sin(30^\circ)} = 2\lambda$$

$\lambda = 200\,m \Rightarrow \boxed{d = 400\,m}$ for neighboring signal. Note that this is true even if the signals from the 2 antennas have a constant phase difference.

(b) For two antennas $I = 4I_0$

For 4 antennas $I = 4^2\left(\frac{I_0}{2}\right) = 8I_0$ which is better. You should explain your reasoning.

(6) (a) $y(x=0,t) = 0$ fixed boundary condition

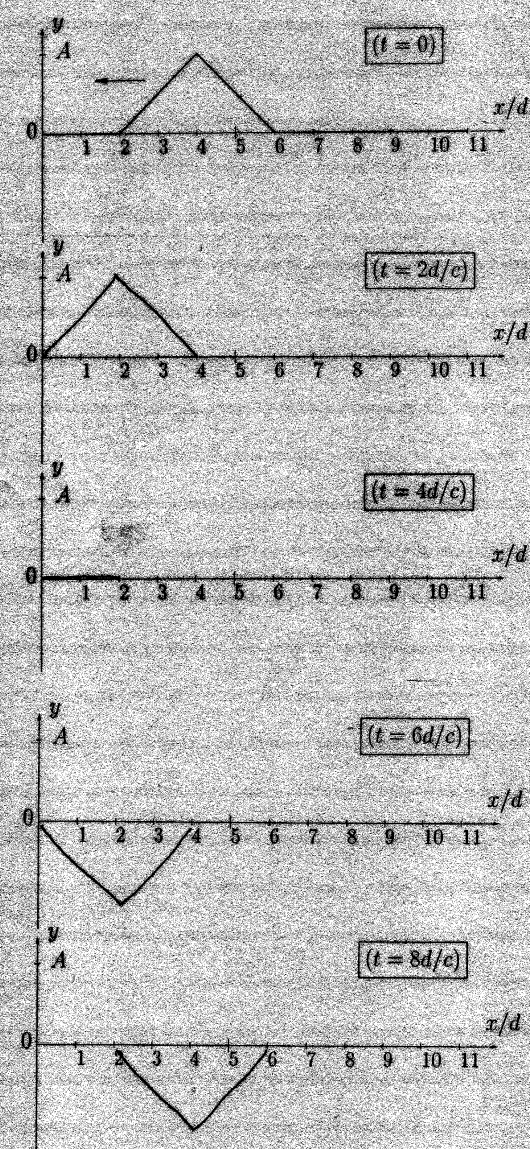
(b) Reflection from fixed boundary condition gives (see formula sheet)

$$y(x,t) = h(x+ct) - h(-(x-ct))$$

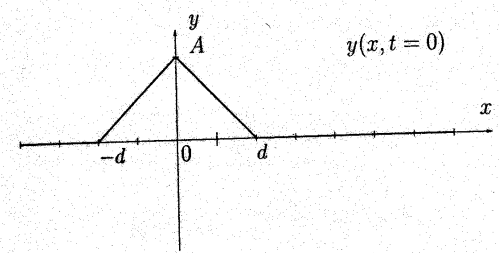
$$= 0 \quad \text{at} \quad x = 0.$$

Since $h(x+ct)$ is given, one sees that the string has the shape as shown:

(6)



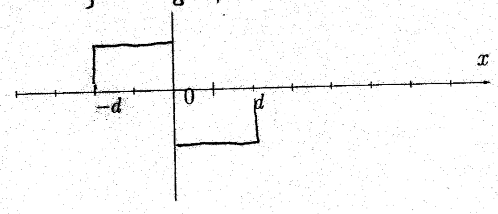
(7)



$$y(x,t) = f(x-ct) + g(x+ct)$$

$$y(x,0) = f(x) + g(x)$$

$y' = f' + g'$, which looks like



Now

$$v_y(x,t=0) = -cf' + cg'$$

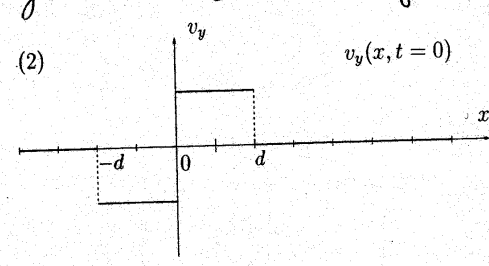
For pure traveling wave, we have either

$$y' = f' \qquad \frac{v_y}{c} = -f' \quad \Rightarrow \quad \frac{v_y}{c} = -y'$$

or

$$y' = g' \qquad \frac{v_y}{c} = g' \quad \Rightarrow \quad \frac{v_y}{c} = y'$$

Clearly (2) is possible for $y = f$, i.e. pure right-moving traveling wave.



(b) For x between 0 and d

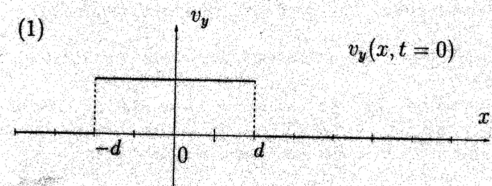
$$y(x, t=0) = A\left(1 - x/d\right)$$

$$y' = -A/d$$

so $\frac{v_y}{c} = -f' = -y' = A/d \qquad v_y = cA/d$

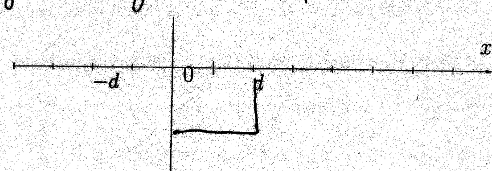
(c) So $v_0 = cA/d$. Now consider

(1)

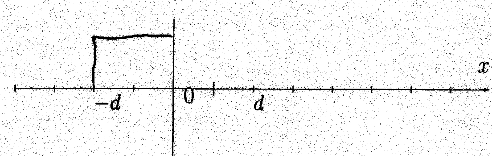


Comparing with $y'(x, t=0)$, we have

f'



and g'



which gives