Physics 214 **Fall 2001**

Problem Set # 1

Reading for coming lectures:
Review *Young & Freedman*, Chapter 13, as necessary.

Problems: (due 9/11)

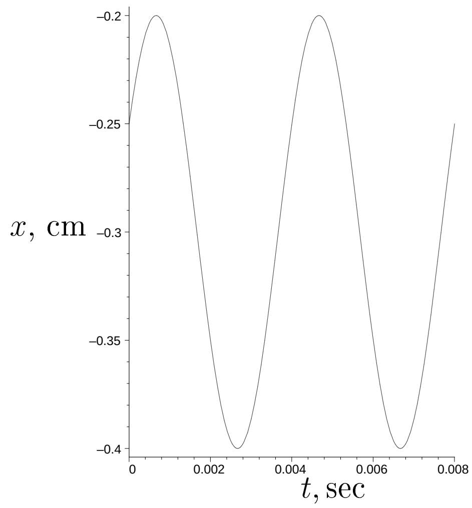
Skills to be mastered:

- *be able to go back and forth between a graph and the values of x_{eq}, A, ω_0, f, T, ϕ_0;*
- *know the difference between the Equation of Motion and a solution of it ;*
- *know how to verify a general solution;*
- *find particular solutions given initial conditions;*

1. Problem 1:

(a) Sketch a graph of $x(t) = x_{\mathrm{eq}} + A\cos(\omega_0 t + \phi_0)$, where $x_{\mathrm{eq}} = 3.0$ cm, $A = +10.0$ cm, $\omega_0 = 6280$ rad/s, and $\phi_0 = -2\pi/3$ rad. Label axes and show at least 2 complete cycles.

(b) Determine the values of x_{eq}, A, ω_0, f, T, and ϕ_0 from the graph of $x(t) = x_{\mathrm{eq}} + A\cos(\omega_0 t + \phi_0)$ shown below.



2. Problem 2:

Which of the following expressions could be a general solution to the equation of motion for an ideal mass-spring system $-k(x - x_{\mathrm{eq}}) = m\dfrac{d^2x}{dt^2}$? In a quick phrase or two, explain under what condition(s) each of the expressions could be a solution or why it cannot be:

(a) $x(t) = x_{\mathrm{eq}} + A \sin \omega_0 t$;

(b) $x(t) = x_{\mathrm{eq}} + A_1 \sin \omega_1 t - A_2 \cos \omega_2 t$;

(c) $x(t) = x_{\mathrm{eq}} + A_1 e^{\omega_1 t} + A_2 e^{-\omega_2 t}$;

(d) $x(t) = x_{\mathrm{eq}} + A \sin [\omega_0 (t - t_0)]$;

(e) $x(t) = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$;

(f) $x(t) = x_{\mathrm{eq}} + B \cos \omega_0 t + C \sin \omega_0 t$;

(g) $x(t) = x_{\mathrm{eq}} - A \sin(\phi_0 - \omega_0 t)$;

(h) $\dfrac{d^2 x}{dt^2} = -\dfrac{k}{m}(x - x_{\mathrm{eq}})$.

3. Problem 3:

A damped oscillator is modeled as a mass m at equilibrium point $x = x_{\mathrm{eq}}$ acted on by (1) an ideal spring of spring constant k and (2) a viscous drag force proportional to the velocity: $\vec{F}_{\mathrm{drag}} = -bm\vec{v}$.

(a) Derive the equation of motion.

(b) Verify that $x(t) = x_{\mathrm{eq}} + Ae^{-bt/2} \sin(\omega' t + \delta_0)$ is a general solution of the equation of motion in the case of small damping $b < \sqrt{4k/m}$. What are the adjustable parameters?

4. Problem 4:

Let $x = 3 + \sqrt{7}i$ and $y = 3 - 4i$. Express the following in both Cartesian form $(a + bi)$ and polar form $(Ae^{i\phi})$:

(a) $x - y$

(b) x^2

(c) $\dfrac{y}{x}$

(d) e^y

5. Problem 5:

An object of mass $m = 0.20$ kg is suspended from the ceiling on a spring of relaxed length 40.0 cm and spring constant $k = 15.0$ N/m. At $t = 0$ the object is at equilibrium and it is given an upward velocity of magnitude 10.0 cm/s. Take $y = 0$ at the ceiling and let the $+y$ axis point downward.

(a) Find the equilibrium position, y_{eq}, of the object.

(b) Write down the equation of motion of the object.

(c) Find the particular solution $y(t)$ to the object's equation of motion for $t \geq 0$.