Physics 214 **Fall 2001**

Problem Set # 2

Reading for coming lectures: *Young & Freedman*, Sections 19.1—19.5 and 20.1—20.4.

Problems: (due 9/18)

Skills to be mastered:

- *be able to determine the* ***complex amplitude*** *from the initial conditions for a simple harmonic oscillator;*
- *be able to get the (real) amplitude and the initial phase from the complex amplitude;*
- *be able to use complex representation to solve differential equations;*
- *be able to convert between wavelength (λ), wavenumber or spatial frequency ($\kappa = 1/\lambda$), and propagation number ($k = 2\pi/\lambda$);*
- *given the wave speed (v) and any one of the quantities (T, f, ω), be able to find any one of (λ, κ, k), and vice versa;*
- *given any one of the quantities (T, f, ω) AND any one of the quantities (λ, κ, k), be able to find v;*
- *be able to distinguish between speed of wave propagation and particle speed.*

1. Problem 1:

An ideal mass-spring system of $k = 160$ N/m and $m = 0.40$ kg has initial position $x_0 = x_{\text{eq}} - 10.0$ cm and initial velocity $v_0 = -40.0$ cm/s.

(a) Find the complex amplitude of the motion.

(b) Find the real amplitude A and the initial phase ϕ_0.

2. Problem 2:

(a) Show, starting from either of the forms $x(t) = x_{\text{eq}} + B\cos(\omega_0 t) + C\sin(\omega_0 t)$ or $x(t) = x_{\text{eq}} + A\cos(\omega_0 t + \phi_0)$, that the *general solution* to the equation of motion of a simple harmonic oscillator (SHO) can be written also as:

$$x(t) = x_{\text{eq}} + \frac{1}{2}\left[\underline{A}e^{i\omega_0 t} + \underline{A}^* e^{-i\omega_0 t}\right] , \tag{1}$$

where $\underline{A}$ is the complex amplitude and $\underline{A}^*$, its complex conjugate.

Hint: Use the fact that for any complex number, $\underline{z}$, it is true that $\Re[\underline{z}] = (\underline{z} + \underline{z}^*)/2$.

(b) Confirm (by explicit differentiation) that (1) satisfies the equation of motion of a SHO.

(c) Express $\underline{A}$ and $\underline{A}^*$ in terms of x_0, v_0, x_{eq}, and ω_0.

3. Problem 3:

(a) Use the complex representation to find a *real general solution* to the equation of motion for the damped harmonic oscillator (see Problem 3, Problem Set #1).

Hint: Try $e^{\underline{\alpha} t}$ where $\underline{\alpha}$ is complex. Be sure to take the real part to get your answer. Compare with the general solution in Problem 3(b), Problem Set #1.

(b) Find an expression for and sketch a graph of the particular solution, if $k = 300$ N/m, $m = 0.40$ kg, $b = 20\ \mathrm{s}^{-1}$, $x_{\mathrm{eq}} = 0$, $x_0 = -6.0$ mm, and $v_0 = -16$ cm/s.

4. Problem 4:

(a) Using the equation of motion for a damped harmonic oscillator, with $x_{\mathrm{eq}} = 0$ for simplicity, show that the time derivative of the total energy, $\frac{dE}{dt}$, at any time is given by

$$\frac{dE}{dt} = -bmv^2\ , \tag{2}$$

where $v = v(t) = \dot{x}(t)$ is the instantaneous velocity of the particle.

(b) Draw sketches of $\frac{dE}{dt}(t)$ and $E(t)$ on the same time scale. Is the behavior of $E(t)$ what you expected? Use physical arguments to justify your answer.

5. Problem 5:

An object of mass m is attached to an ideal spring (k) and driven by an external force $F_{\mathrm{ext}}(t) = F_0 \cos(\omega t)$. The drag force exerted on the object is given by $F_{\mathrm{drag}} = -cm\frac{d^4x}{dt^4}$.

(a) Find the (real) amplitude A of oscillations.

Hint: As in class, assume that the oscillations are harmonic and use the complex representation.

(b) Sketch A/f versus ω, where $f \equiv F_0/m$. (*Note:* f has units of force/mass and is not to be confused with frequency!)