

3) a) equation of motion: $-k(x - x_{eq}) - b m \frac{dx}{dt} = m \frac{d^2x}{dt^2}$

try solution $x(t) = x_{eq} + e^{\alpha t}$

$$-k(x_{eq} + e^{\alpha t} - x_{eq}) - b m (\alpha e^{\alpha t}) = m (\alpha)^2 e^{\alpha t}$$

$$-\frac{k}{m} (e^{\alpha t}) - b (\alpha e^{\alpha t}) - (\alpha)^2 e^{\alpha t} = 0$$

$$(\alpha)^2 + b\alpha + \frac{k}{m} = 0$$

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4(\frac{k}{m})}}{2}$$

$\therefore$ this is a solution if $\alpha = \frac{-b \pm \sqrt{b^2 - 4\frac{k}{m}}}{2}$

A general solution is a linear combination of these 2 solutions

$$x(t) = x_{eq} + \underline{A} e^{\alpha_1 t} + \underline{B} e^{\alpha_2 t}$$

where $\alpha_1 = \frac{-b + \sqrt{b^2 - 4\frac{k}{m}}}{2}$ and $\alpha_2 = \frac{-b - \sqrt{b^2 - 4\frac{k}{m}}}{2}$

b) $\alpha_1 = \frac{-20 + \sqrt{(20)^2 - 4(\frac{300}{.40})}}{2} = -10 + 25.5i$

$\alpha_2 = -10 - 25.5i \Rightarrow x(t) = \underline{A} e^{(-10+25.5i)t} + \underline{B} e^{(-10-25.5i)t}$

$x_0 = x(t=0) = \underline{A} e^{(-10+25.5i)0} + \underline{B} e^{(-10-25.5i)0} = \underline{A} + \underline{B} = -6 \times 10^{-3}$
 $\underline{B} = -6 \times 10^{-3} - \underline{A}$

$v_0 = v(t=0) = (-10+25.5i) \underline{A} e^{(-10+25.5i)0} + (-10-25.5i) \underline{B} e^{(-10-25.5i)0} =$

$= (-10+25.5i) \underline{A} + (-10-25.5i) \underline{B} = -16$

$= (-10+25.5i) \underline{A} + (-10-25.5i) (-6 \times 10^{-3} - \underline{A}) = -16$