5) continued

substitute these back into eq. of motion:

$$-K\left(Re\left[\underline{A}e^{i\omega t}\right]\right) - cm\, Re\left[\omega^4 \underline{A}\, e^{i\omega t}\right] + F_o \cos(\omega t) = m\, Re\left[-\omega^2 \underline{A}\, e^{i\omega t}\right]$$

now write $F_o \cos\omega t$ as $Re\left[F_o e^{i\omega t}\right]$ using Euler's Identity

and divide by m, let $\omega_o = \sqrt{\frac{K}{m}}$

$$-\omega_o^2 Re\left[\underline{A}e^{i\omega t}\right] - c\omega^4 Re\left[\underline{A}e^{i\omega t}\right] + \omega^2 Re\left[\underline{A}e^{i\omega t}\right] + Re\left[+\frac{F_o}{m} e^{i\omega t}\right] = 0$$

$$Re\left[e^{i\omega t}\left(\underline{A}\left[-\omega_o^2 - c\omega^4 + \omega^2\right] + \frac{F_o}{m}\right)\right] = 0$$

which works only if $\underline{A}\left[-\omega_o^2 - c\omega^4 + \omega^2\right] + \frac{F_o}{m} = 0$

$$\Rightarrow \underline{A} = \frac{-F_o/m}{\left[-\omega_o^2 - c\omega^4 + \omega^2\right]} = \frac{F_o/m}{\left[\omega_o^2 + c\omega^4 - \omega^2\right]}$$

want to write $\underline{A}$ as $\underline{A} = Ae^{i\varphi}$

$$\boxed{A = |\underline{A}| = \frac{F_o/m}{|\omega_o^2 + c\omega^4 - \omega^2|}}$$

because numerator ~~and~~ and denominator are both real

Since $\underline{A}$ is always REAL, φ is either 0 (when $\underline{A}$ is positive) or π (when $\underline{A}$ is negative).