We can then find an expression for $v_y(x,t)$ by taking the partial derivative of $y(x,t)$ with respect to TIME and evaluating it at (x,t):

$$v_y(x,t) = \frac{\partial y}{\partial t}(x,t)$$

$$= \frac{\partial}{\partial t}\left[-A\sin\frac{2\pi}{\lambda}(x-vt)\right]$$

$$= +\frac{2\pi}{\lambda}\cdot v\cdot A\cos\left[\frac{2\pi}{\lambda}(x-vt)\right]$$

$$= 2\pi f A\cos\left[\frac{2\pi}{\lambda}(x-vt)\right] = \omega A\cos\left[\frac{2\pi}{\lambda}(x-vt)\right]$$

(c) The MAGNITUDE of $v_y(x,t)$ is:

$$|v_y(x,t)| = \frac{2\pi}{\lambda}\cdot v\cdot A\,\underbrace{\left|\cos\left[\frac{2\pi}{\lambda}(x-vt)\right]\right|}_{\leq 1} \leq \frac{2\pi}{\lambda}\cdot v\cdot A$$

$$\Rightarrow |v_y|_{max} = \frac{2\pi A}{\lambda}\cdot v$$

1) $|v_y|_{max} = v$, iff $A = \frac{\lambda}{2\pi}$

2) $|v_y|_{max} < v$, iff $A < \frac{\lambda}{2\pi}$

3) $|v_y|_{max} > v$, iff $A > \frac{\lambda}{2\pi}$