Also, $|\vec{F}_{sou\,w}| = \tau$, and therefore

$$\begin{cases} F_{sou\,w,x} = \tau\cos\theta \\ F_{sou\,w,y} = \tau\sin\theta \end{cases}$$

However, since $kA \ll 1 \Rightarrow \frac{2\pi}{\lambda} A \ll 1$

$$\Rightarrow A \ll \frac{\lambda}{2\pi} \lesssim L$$

$$\Rightarrow \underline{\underline{A \ll L \Rightarrow \theta \ll 1}}$$

Using $\begin{cases} \cos\theta = 1 - \frac{\theta^2}{2} + \cdots \approx 1 \\ \sin\theta = \theta - \frac{\theta^3}{6} + \cdots \approx \theta \approx \tan\theta \end{cases}$, for $\underline{\theta \ll 1}$

$\Rightarrow$

(a) $$\underline{\underline{F_{sou\,w,x} = \tau\cos\theta \approx \tau = \left(\frac{\omega}{k}\right)^2 \frac{m}{L}}} \quad (5a)$$

(b) $$F_{sou\,w,y} = \tau\sin\theta \approx \tau\tan\theta = \tau \frac{\partial y}{\partial x}(x=0,t)$$
$$= \left(\frac{\omega}{k}\right)^2 \frac{m}{L} \left(kA\cos(kx)\cos(\omega t)\big|_{x=0}\right)$$