

③

$$\bullet \lim_{\omega \rightarrow \infty} \tan \phi(\omega) = \lim_{\omega \rightarrow +\infty} \left[\frac{b\omega}{\omega^2 - \omega_0^2} \right] = 0$$

$$\Rightarrow \phi(\omega) = 0 \text{ or } \pm\pi,$$

but now

$$\lim_{\omega \rightarrow +\infty} \cos \phi(\omega) = \lim_{\omega \rightarrow \infty} \frac{\omega_0^2 - \omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (b\omega)^2}} = -1$$

$$\Rightarrow \underline{\underline{\lim_{\omega \rightarrow \infty} \phi(\omega) = \pm\pi}}$$

(c) We are now going to study where the function $(A/f)(\omega)$ has MINIMA or MAXIMA. As suggested in the hint, since

$$(A/f)(\omega) = \frac{1}{\sqrt{D(\omega)}}; \quad D(\omega) \equiv (\omega_0^2 - \omega^2)^2 + (b\omega)^2,$$

it is enough to know the MAXIMA/MINIMA of $D(\omega)$:

$$\begin{aligned} \bullet \frac{dD}{d\omega} &= 2(\omega_0^2 - \omega^2)(-2\omega) + 2b\omega \cdot b \\ &= -2\omega[2\omega_0^2 - 2\omega^2 - b^2] = 0 \end{aligned}$$

$$\Rightarrow \frac{dD}{d\omega} \text{ VANISHES for } \boxed{\omega = 0, \pm \sqrt{\omega_0^2 - b^2/2}}$$