Physics 214 **Fall 2001**

Problem Set # 4

Reading for coming lectures: *Young & Freedman*, Sections 21.1—21.3; 33.1—33.4; review the relevant material from electromagnetism as necessary.

Problems: (due 10/02)

Skills to be mastered:

- *being able to relate the wave speed (v) to the physical parameters of the system in which the waves are propagating: i.e., $v = c \equiv \sqrt{\tau/\mu}$ for a string; $v = c \equiv \sqrt{B/\rho}$ for sound;*
- *given the wave solution $y(x,t)$, be able to get and interpret "snapshots" $y(x, t = t_0)$ and "particle histories" $y(x = x_0, t)$;*
- *understand the boundary conditions for both fixed and free ends of a string;*
- *be able to sketch the allowed modes for different types of boundary conditions;*
- *be able to pick the wavelength off of pictures of different standing wave modes;*
- *be able to get the allowed frequencies from the allowed wavelengths;*

1. Problem 1:
A transverse wave on a string is described by $y(x,t) = \mathfrak{Re}\left\{\underline{A}e^{i(kx-\omega_0 t)}\right\}$, where $\underline{A} = (4 - 3i)$ mm, $k = 12.56m^{-1}$, and $\omega_0 = 6280s^{-1}$.

(a) Sketch a *snapshot* of the string at $t = 0.5$ ms (i.e., a graph of $y(x)$ at $t = 0.5$ ms).

(b) Sketch a *particle history* for the particle at $x = -50$ cm (i.e. a graph of $y(t)$ at $x = -50$ cm).

2. Problem 2:
A string of length L is under tension τ and has linear mass density μ. The string is fixed at one end ($x = L$) while the other end ($x = 0$) is attached to a massless ring that can slide freely (without friction) up and down a rod. Attached to the ring is an ideal spring that exerts a vertical force on the ring; when the ring is at $y = 0$ the spring is relaxed. (See Figure 1.) (In this problem, $K_{\rm spr}$ represents the spring constant to avoid confusion with the wave-vector $k = 2\pi/\lambda$.)

(a) Use $\vec{F} = m\vec{a}$ to find a relationship between the slope of the string at $x = 0$ and the displacement $y(x = 0, t)$ of that point (which is the same as the displacement of the ring).

(b) In the limit $K_{\rm spr} \to \infty$, the point $x = 0$ acts like a *fixed* end. What does your answer to part (a) tell you about the motion and/or the shape of the string at $x = 0$ in that limit? Check that a standing wave on the string consistent with the boundary condition for $K_{\rm spr} = \infty$ must have the form

$$y(x,t) = A\sin(kx)\cos(\omega t) \tag{1}$$

where $k = 2\pi/\lambda$. (Assume $A \ll \lambda$.) What are the allowed values of λ (i.e., the *normal modes*) in terms of the string length L, for a standing wave (1) to be formed on the string?

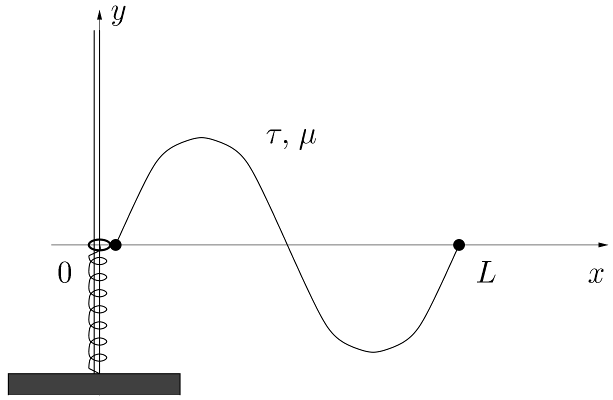

Figure 1: Generalized boundary conditions on a string.

(c) In the limit $K_{\text{spr}} \to 0$, the point $x = 0$ acts like a *free* end. What does your answer to (a) tell you about the motion and/or the shape of the string at $x = 0$ in that limit? Check that a standing wave on the string consistent with the boundary condition for $K_{\text{spr}} = 0$ must have the form

$$y(x,t) = A\cos(kx)\cos(\omega t) \tag{2}$$

where $k = 2\pi/\lambda$. (Assume $A \ll \lambda$.) What are the allowed values of λ (i.e., the *normal modes*) in terms of the string length L, for a standing wave (2) to be formed on the string?

(d) Now let the spring constant K_{spr} have an *arbitrary* value. Would any of the standing wave solutions (1) or (2) work for the general boundary condition you derived in (a)? Why not?

Argue that a standing wave of the form

$$y(x,t) = A\cos(kx + \phi')\cos(\omega t) \tag{3}$$

would work in this case. Find the condition(s) that the initial phase ϕ' and the wavelength λ should satisfy if a standing wave (3) is to be formed on the string.

(e) Check that the condition(s) you derived in part (d) would give you the same results for the allowed values of λ as parts (b) and (c) in the limits $K_{\text{spr}} \to \infty$ and $K_{\text{spr}} \to 0$. (*Don not try to solve the conditions for λ in the general case!*)

3. Problem 3:

The E-string of a violin is 33 cm long (between the two fixed points) and has diameter 0.25 mm. It is made of steel ($\rho = 7800$ kg/m^3).

(a) If its fundamental frequency is 660 Hz, what is the tension τ?

(b) Consider the chunk of the string between the points $x = 16$ cm and $x + \Delta x = 17$ cm (see Figure 2). What is the net force acting on the chunk ignoring gravity? What its direction? Express your answer in terms of the degree of freedom $y(x,t)$ and/or its derivatives and the string tension τ.

(c) How does the *maximum net force* compare with the force of gravity acting on the chunk? Were we justified to neglect it in part (b)? [*Hint:* Use the standing wave solution (1) derived in Problem 2.]

(d) What is the *maximum net force* exerted by the string on the peg at its end?

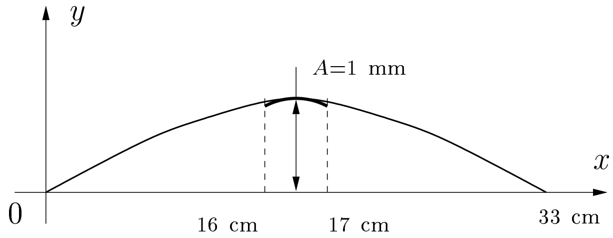

Figure 2: Waves on a violin E-string.

(e) The violinist can make the string vibrate in a higher frequency mode (without any of the lower frequency modes) by touching the string lightly at a *node*; the string vibrates on both sides of this point. This is called playing a harmonic. If the violinist wishes to play a harmonic at 3960 Hz (without any lower frequency modes), how far from the end of the string should she touch it? Sketch the resulting standing wave pattern and indicate where the violinist is touching the string.

(f) If instead she presses firmly at this same point, effectively shortening the length of the string, what fundamental frequency results? (Note that string is divided into two unequal lengths; the part of the string that is free to vibrate is the longer of the two.)

4. Problem 4:

Young & Freedman, Problem 19-15.

5. Problem 5:

Young & Freedman, Problem 19-23.

6. Problem 6:
Two closed cylindrical tubes are filled respectively with neon (Ne) and argon (Ar), both at atmospheric pressure. Assuming the gases in both tubes vibrate in their fundamental modes and that the bulk moduli of Ne and Ar are the same, *how much shorter* should the Ar-tube be in order to produce the *same tone* as the Ne-tube?

7. Problem 7:
A simple model of an oboe (a double-reed woodwind instrument) is a pipe with one closed end (the reed end, a pressure anti-node) and one open end (a pressure node). One particular standing wave in an oboe has the pressure pattern shown on Figure 3:

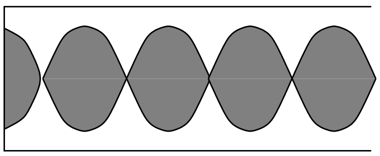

Figure 3: Standing waves in an oboe.

The length of the oboe is 75 cm and the speed of sound is 350 m/s. What is the frequency of the standing wave?