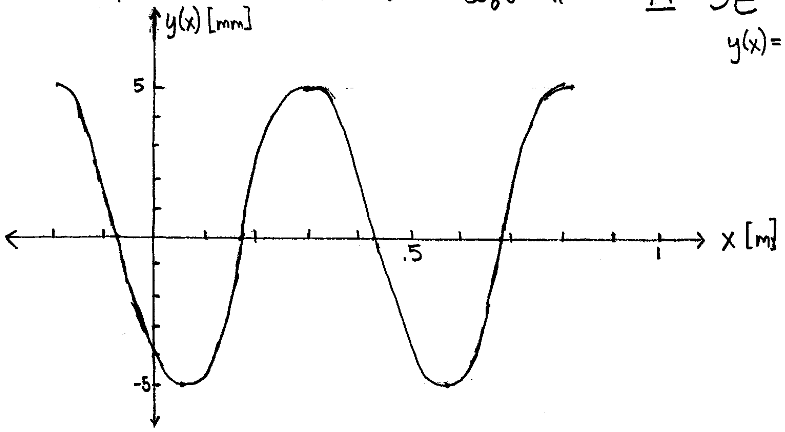
Problem Set #4 — Solutions

1a. Snapshot at $t = 0.5\,ms$ $\quad \omega_0 t = \pi \quad \underline{A} = 5e^{-.205\pi i}$

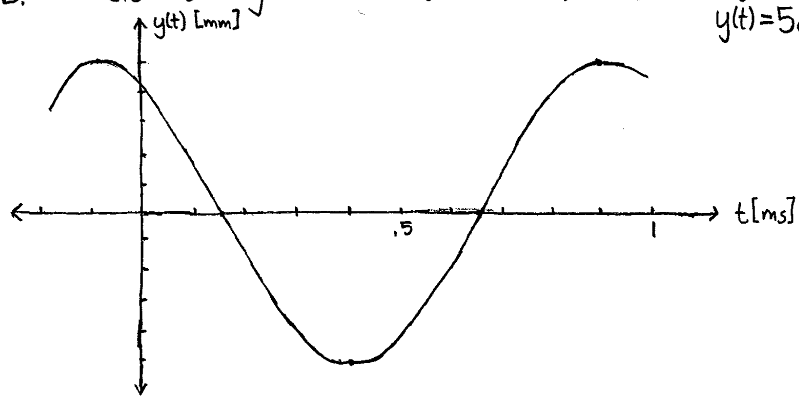
$y(x) = -5\cos 2\pi(2x - .1)$



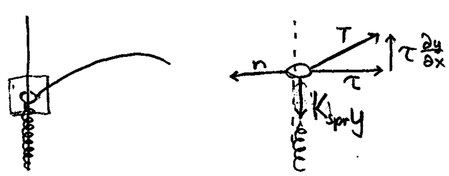
b. Particle history at $x = -0.5\,m \quad kx = -2\pi$

$\omega_0 = 2\pi(1000)\,\frac{rad}{s} = 2\pi\,\frac{rad}{ms}$

$y(t) = 5\cos 2\pi(t + .1)$



2a.



$F_y = ma_y \quad m = 0$ massless ring

$\tau \frac{\partial y(0,t)}{\partial x} - K_{spr}\, y(0,t) = 0$

$\tau \frac{\partial y(0,t)}{\partial x} = K_{spr}\, y(0,t)$

b. $K_{spr} \to \infty \quad y(0,t) = \frac{\tau}{K_{spr}} \frac{\partial y}{\partial x} = 0 \quad$ Other BC is $y(L,t) = 0$

Try $y(x,t) = A\sin(kx)\cos(\omega t) \Rightarrow y(0,t) = A\sin 0 \cos(\omega t) = 0$

$y(L,t) = A\sin(kL)\cos(\omega t) = 0 \Rightarrow kL = n\pi \quad n = 1,2,3\ldots$

$\lambda = \frac{2\pi}{k} = \frac{2L}{n} \quad \lambda = 2L, L, \frac{2}{3}L, \ldots$

c. $K_{spr} \to 0 \quad \frac{\partial y(0,t)}{\partial x} = \frac{K_{spr}}{\tau} y(0,t) = 0 \quad$ Other BC is $y(L,t) = 0$

Try $y(x,t) = A\cos(kx)\cos(\omega t) \Rightarrow \frac{\partial y(0,t)}{\partial x} = -Ak\sin 0 \cos(\omega t) = 0$

$y(L,t) = A\cos(kL)\cos(\omega t) = 0 \Rightarrow kL = \pi(n - \frac{1}{2}) \quad n = 1,2,3,\ldots$

2c (cont.) $\lambda = \frac{2\pi}{k} = \frac{2L}{n-\frac{1}{2}}$ $n=1,2,\ldots$ $\lambda = 4L, \frac{4}{3}L, \frac{4}{5}L$

d. The BC $\tau \frac{\partial y(0,t)}{\partial x} = K_{spr} y(0,t)$ is not satisfied by either (1) or (2) because now the position can only be zero iff the slope is zero.

Try $y(x,t) = A\cos(kx+\phi)\cos(\omega t)$ $y(0,t) = A\cos\phi'\cos(\omega t)$ $\frac{\partial y(0,t)}{\partial x} = -Ak\sin\phi'\cos(\omega t)$

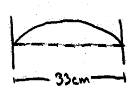
$(K_{spr}\cos\phi')\cancel{A\cos(\omega t)} = (-k\tau\sin\phi')\cancel{A\cos\omega t} \Rightarrow \phi' = \tan^{-1}\left(-\frac{K_{spr}}{k\tau}\right)$

The other BC $y(L,t) = A\cos(kL+\phi')\cos(\omega t) = 0 \Rightarrow kL + \phi' = \pi(n-\frac{1}{2})$ $n=1,2,\ldots$

$\lambda = \frac{2\pi}{k} = \frac{2L}{n-\frac{1}{2}-\phi'/\pi}$

e. $K_{spr} \to \infty$ $\phi' = \tan^{-1}(-\infty) = -\frac{\pi}{2}$ $\lambda = \frac{2L}{n-\frac{1}{2}+\frac{1}{2}} = \frac{2L}{n}$ $n=1,2,\ldots$

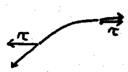
$K_{spr} \to 0$ $\phi' = \tan^{-1}(0)$ $\lambda = \frac{2L}{n-\frac{1}{2}+0} = \frac{2L}{n-\frac{1}{2}}$ $n=1,2,\ldots$

3a. 

$\mu = \rho(\pi r^2) = (7800\,\frac{kg}{m^3})\frac{(\pi)}{(4)}(2.5\times10^{-4}\,m)^2 = 3.83\times10^{-4}\,\frac{kg}{m}$

$f_0 = \frac{\omega}{2\pi} = \frac{vk}{2\pi}$ $k_0 = \frac{\pi}{L} \Rightarrow v = 2f_0 L = 435.6\,\frac{m}{s}$

$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = \mu v^2 = 72.7\,N$

b. 

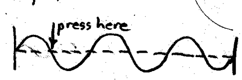
$F = \tau\frac{\partial y(17,t)}{\partial x} - \tau\frac{\partial y(16,t)}{\partial x}$ Direction is purely vertical, downward for chunk shown in Fig 2.

c. $\frac{\partial y(x,t)}{\partial x} = Ak\cos(kx)\cos(\omega t)$ $A = 1\times10^{-3}\,m$ $k = \frac{\pi}{L} = 9.52\,\frac{rad}{m}$

$F_{max} = \tau(Ak)\left|\cos\frac{17\pi}{33} - \cos\frac{16\pi}{33}\right| = 0.065\,N$

$F_{grav} = mg$ $m = \mu(1cm)$ $\therefore$ $F_{grav} = 1.5\times10^{-4}\,N \ll F_{max}$ We can neglect F_{grav}.

d. Net force at $x = 33\,cm$ $F_{end,max} = \tau(Ak)\left|\cos\frac{33\pi}{33}\right| = 0.7\,N$

e. $3960\,Hz = 6(660\,Hz)$ 

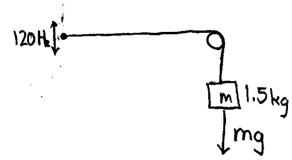
Violinist should press $\frac{1}{6}$th from the end (5.5 cm).

f. $L' = \frac{5}{6}L = 27.5\,cm$ $f_0' = \frac{v}{2L'} = 792\,Hz$ $f_0' = \frac{6}{5}f_0$

The resulting note is a G, which is $\frac{6}{5}$th the frequency of the E. This can be seen from the harmonic series:

C	C	G	C	E	G	...
1	2	3	4	5	6	

4) Y&F, Prob 19-15



a. $\mu = 5.5\times10^{-2}\ \frac{kg}{m}$ $\quad f = 120\ Hz$

$\tau = 1.5\ kg \cdot 9.8\ \frac{m}{s^2} = 14.7\ N$ force of gravity

$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{14.7}{5.5\times10^{-2}}} = 16.3\ m/s$ speed of a transverse wave

b. $\lambda f = v$ dispersion relation

$\lambda = \frac{v}{f} = \frac{16.3}{120} = 13.6\ cm$ wavelength

c. If mass were 3.0 kg, the tension τ would double. Then the speed and wavelength would increase by a factor of $\sqrt{2}$.

5) Y&F, Prob 19-23

a. $0.85\ v_{sound} = 850\ \frac{km}{h} \Rightarrow v_{sound} = 1000\ \frac{km}{h}$

$v_s(t) = \sqrt{\frac{\gamma RT}{M}}$ speed of sound at a given temperature

At 20°C (293°K), speed of sound is $344\ m/s$.

$344 = \sqrt{\frac{\gamma R}{n}}\sqrt{293} \Rightarrow \sqrt{\frac{\gamma R}{M}} = 20.1$

At altitude, $v_s(T) = \sqrt{\frac{\gamma R}{M}}\sqrt{T} = 1000\ \frac{km}{h}\left(\frac{1000\ m}{1\ km}\right)\left(\frac{1\ h}{3600\ s}\right) = 278\ \frac{m}{s}$

$T = \left(\frac{278}{20.1}\right)^2 = 191°K = -82°C$

b. The other equation for sound speed involves both pressure and density of the air. Without the density, we cannot find the pressure at this altitude.

6. Molar masses: $M_{Ne} = 20.180$ $\quad M_{Ar} = 39.948$

$$v = \sqrt{\frac{\gamma RT}{M}} \qquad \frac{v_{Ar}}{v_{Ne}} = \sqrt{\frac{M_{Ne}}{M_{Ar}}} = .71 \qquad \frac{\lambda_{Ar}}{\lambda_{Ne}} = \frac{v_{Ar}}{v_{Ne}}\frac{\cancel{f}}{\cancel{f}} \Rightarrow .71$$

The Ar-tube should be 71% of the length of the Ne-tube.

7. There are $9/4$ of a wave inside the oboe.

$$k = \frac{2\pi}{(.75\,m)}\left(\frac{9}{4}\right) = 6\pi\ \frac{rad}{m}$$

$$f = \frac{\omega}{2\pi} = \frac{v}{2\pi}k = 3(350\ m/s) = 1050\ Hz$$