(1.) Young & Freedman 33-5

$\vec{E}(y,t) = -(3.10\times10^5 \text{ V/m})\hat{k}\sin[(2.65\times10^{12}\text{ rad/s})t - ky]$

(a) In what direction is the wave traveling

Form: $\vec{E} = E_{max}\sin[\omega t - ky] \Rightarrow$ Wave is traveling in +y direction.

(b) What is the wavelength of the wave?

$c = \frac{\omega}{k}$ $\quad c = 3.0\times10^8 \text{ m/s}$ $\quad \omega = 2.65\times10^{12}\text{ rad/s}$ $\quad k = \frac{2\pi}{\lambda}$

$3.0\times10^8 \text{ m/s} = (2.65\times10^{12}\text{ rad/s})\frac{\lambda}{2\pi}$

$\lambda = 2\pi \frac{3.0\times10^8 \text{ m/s}}{2.65\times10^{12}\text{ rad/s}}$ $\quad \boxed{\lambda = 7.11\times10^{-4}\text{ m}}$

(c) Write the vector equation for $\vec{B}(y,t)$.

$B_{max} = \frac{E_{max}}{c} = \frac{-3.10\times10^5 \text{ V/m}}{3.0\times10^8 \text{ m/s}} = -1.03\times10^{-3}\text{ V}\cdot\text{s/m}^2 = -1.03\times10^{-3}\text{ T}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.11\times10^{-4}\text{ m}} = 8.84\times10^3 \text{ rad/m}$

$\vec{B} \perp \vec{E}$, $\vec{B} \perp \vec{S}$ (direction of propagation)

$\vec{B}(y,t) = (-1.03\times10^{-3}\text{ T})\hat{\imath}\sin[(2.65\times10^{12}\text{ rad/s})t - (8.84\times10^3\text{ rad/m})y]$

(2) Young & Freedman 33-24

Conducting Planes act as fixed ends for $\vec{E} \Rightarrow$ see p. 1042 Y&F

$f = 750$ MHz $\quad L = 80.0$ cm $\quad c = \lambda f$

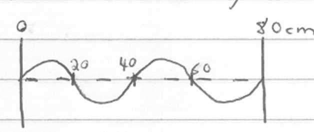
$3.0\times10^8 \text{ m/s} = \lambda(750\times10^6 \frac{1}{s})$

$\lambda = 0.4\text{ m} = 40\text{ cm}$

A point charge can remain at rest at points where $\vec{E}$ is always zero. This occurs at nodes of the $\vec{E}$ wave.

Conducting Planes $\Rightarrow$ 2 fixed ends

$L = 80$ cm, $\lambda = 40$ cm $\Rightarrow$ 2 full cycles



The three internal nodes are at 20cm, 40cm, 60cm.

$\Rightarrow$ A point charge can be placed at rest and remain at rest at 20.0cm, 40.0cm, or 60.0cm

(3) Young & Freedman 33-30

$\vec{E} = E_{max}\,\hat{j}\,\sin(\omega t - kx)$ $\vec{B} = B_{max}\,\hat{k}\,\sin(\omega t - kx + \phi)$ $-\pi \le \phi \le \pi$

Show $E_{max} = cB_{max}$, $\phi = 0$

(33-12): $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$ (33-14): $-\frac{\partial B_z}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t}$

$$\frac{\partial E_y}{\partial x} = -kE_{max}\sin(\omega t - kx)$$

$$\frac{\partial B_z}{\partial t} = \omega B_{max}\sin(\omega t - kx + \phi)$$

(33-12): $-kE_{max}\sin(\omega t - kx) = -\omega B_{max}\sin(\omega t - kx + \phi)$

sine terms must be equal, so $\boxed{\phi = 0}$

amplitudes must be equal, so $-kE_{max} = -\omega B_{max}$

$$E_{max} = \frac{\omega}{k} B_{max}$$

$\frac{\omega}{k} = c \implies \boxed{E_{max} = cB_{max}}$

(4) (a) Show that the expression $\vec{E}(x,t) = \hat{y}E_0\cos(\omega t - kx) - \hat{z}E_0\sin(\omega t - kx)$ is a solution to the wave equation for $\vec{E}(x,t)$.

$$\frac{\partial^2 E_z}{\partial t^2} = c^2 \frac{\partial^2 E_z}{\partial x^2}$$

$\frac{\partial E_z}{\partial t} = -E_0\omega\cos(\omega t - kx)$ $\frac{\partial E_z}{\partial x} = E_0 k\cos(\omega t - kx)$

$\frac{\partial^2 E_z}{\partial t^2} = E_0\omega^2\sin(\omega t - kx)$ $\frac{\partial^2 E_z}{\partial x^2} = E_0 k^2\sin(\omega t - kx)$

$$E_0\omega^2\sin(\omega t - kx) = c^2(E_0k^2\sin(\omega t - kx))$$

$$\omega^2 = c^2k^2 \quad c^2 = \frac{\omega^2}{k^2} \quad \frac{\omega}{k} = c \ \checkmark$$

$$\frac{\partial^2 E_y}{\partial t^2} = c^2 \frac{\partial^2 E_y}{\partial x^2}$$

$\frac{\partial E_y}{\partial t} = -E_0\omega\sin(\omega t - kx)$ $\frac{\partial E_y}{\partial x} = E_0 k\sin(\omega t - kx)$

$\frac{\partial^2 E_y}{\partial t^2} = -E_0\omega^2\cos(\omega t - kx)$ $\frac{\partial^2 E_y}{\partial x^2} = -E_0k^2\cos(\omega t - kx)$

$$-E_0\omega^2\cos(\omega t - kx) = c^2(-E_0k^2\cos(\omega t - kx))$$

$$\omega^2 = c^2k^2 \quad c^2 = \frac{\omega^2}{k^2} \quad \frac{\omega}{k} = c \ \checkmark$$

(b) Show that $\vec{E}(x,t) = \hat{y}E_0\cos(\omega t - kx) - \hat{z}E_0\sin(\omega t - kx)$ can be written as $\vec{E}(x,t) = Re[(\hat{y}+i\hat{z})E_0 e^{i(\omega t-kx)}]$

$$\vec{E}(x,t) = \hat{y}E_0\cos(\omega t - kx) - \hat{z}E_0\sin(\omega t - kx)$$
$$= \hat{y}E_0 Re[e^{i(\omega t-kx)}] - \hat{z}E_0 Im[e^{i(\omega t-kx)}]$$

use $Im[A] = Re[-iA]$

$$\vec{E}(x,t) = \hat{y}E_0 Re[e^{i(\omega t-kx)}] - \hat{z}E_0 Re[-ie^{i(\omega t-kx)}]$$
$$= Re[\hat{y}E_0 e^{i(\omega t-kx)}] + Re[i\hat{z}E_0 e^{i(\omega t-kx)}]$$
$$\boxed{\vec{E}(x,t) = Re[(\hat{y}+i\hat{z})E_0 e^{i(\omega t-kx)}]}$$

(c) Find $\vec{B}(x,t)$ in real and complex representation.

Eq. (d) from 10-2-01 lecture: $\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}$

$$\frac{\partial B_y}{\partial t} = E_0 k\cos(\omega t - kx)$$
$$B_y = \int E_0 k\cos(\omega t-kx)\,dt$$
$$B_y = E_0\frac{k}{\omega}\sin(\omega t-kx) + C \qquad C=0 \text{ because } B_{eq} \text{ is at } y=0$$
$$\underline{B_y = \frac{E_0}{c}\sin(\omega t-kx)}$$

Eq. (b) from 10-2-01 lecture: $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$

$$\frac{\partial B_z}{\partial t} = -E_0 k\sin(\omega t-kx)$$
$$B_z = -\int E_0 k\sin(\omega t-kx)\,dt$$
$$B_z = +E_0\frac{k}{\omega}\cos(\omega t-kx) + C \qquad C=0 \text{ because } B_{eq} \text{ is at } z=0$$
$$\underline{B_z = +\frac{E_0}{c}\cos(\omega t-kx)}$$

$$\boxed{\vec{B}(x,t) = \hat{y}E_0\frac{1}{c}\sin(\omega t-kx) + \hat{z}\frac{E_0}{c}\cos(\omega t-kx)}$$

$$= \hat{y}\frac{E_0}{c} Im[e^{i(\omega t-kx)}] + \hat{z}\frac{E_0}{c} Re[e^{i(\omega t-kx)}]$$
$$= Re[-i\hat{y}\frac{E_0}{c}e^{i(\omega t-kx)}] + Re[+\hat{z}\frac{E_0}{c}e^{i(\omega t-kx)}]$$

$$\boxed{\vec{B}(x,t) = Re\left[(-i\hat{y}+\hat{z})\frac{E_0}{c}e^{i(\omega t-kx)}\right]}$$

(d) $\boxed{t=0}$: $\vec{E}(x,0) = \hat{y}E_0\cos(-kx) - \hat{z}E_0\sin(-kx)$

$= \hat{y}E_0\cos kx + \hat{z}E_0\sin kx$

$x = 0, \frac{\lambda}{4}, \frac{\lambda}{2}, \frac{3\lambda}{4}, \lambda$

$\underline{\vec{E}(0,0) = \hat{y}E_0}$ $\quad \vec{E}(\frac{\lambda}{4},0) = \hat{y}E_0\cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right) + \hat{z}E_0\sin\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right)$

$\underline{\vec{E}(\frac{\lambda}{4},0) = \hat{z}E_0}$

$\vec{E}(\frac{\lambda}{2},0) = \hat{y}E_0\cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right) + \hat{z}E_0\sin\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right)$

$\underline{\vec{E}(\frac{\lambda}{2},0) = -\hat{y}E_0}$

$\vec{E}(\frac{3\lambda}{4},0) = \hat{y}E_0\cos\left(\frac{2\pi}{\lambda}\frac{3\lambda}{4}\right) + \hat{z}E_0\sin\left(\frac{2\pi}{\lambda}\frac{3\lambda}{4}\right)$

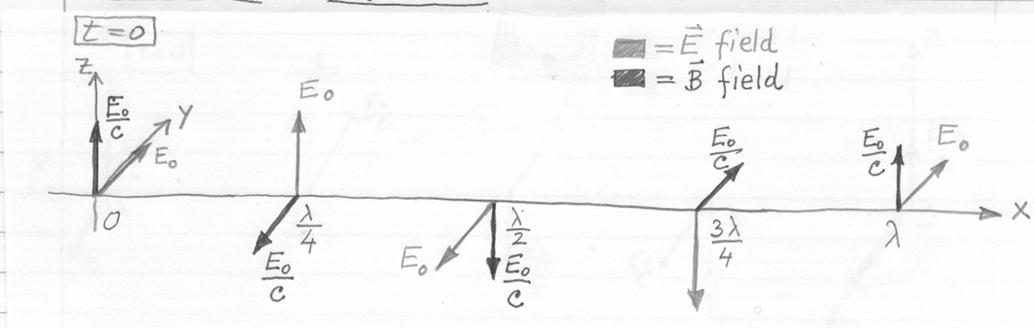
$\underline{\vec{E}(\frac{3\lambda}{4},0) = -\hat{z}E_0}$ $\quad \underline{\vec{E}(\lambda,0) = \hat{y}E_0}$

$\vec{B}(x,0) = \hat{y}\frac{E_0}{c}\sin(-kx) + \hat{z}\frac{E_0}{c}\cos(-kx)$

$= -\hat{y}\frac{E_0}{c}\sin(kx) + \hat{z}\frac{E_0}{c}\cos(kx)$

$\underline{\vec{B}(0,0) = +\hat{z}\frac{E_0}{c}}$ $\quad \underline{\vec{B}(\frac{\lambda}{4},0) = -\hat{y}\frac{E_0}{c}}$ $\quad \underline{\vec{B}(\frac{\lambda}{2},0) = -\hat{z}\frac{E_0}{c}}$

$\underline{\vec{B}(\frac{3\lambda}{4},0) = \hat{y}\frac{E_0}{c}}$ $\quad \underline{\vec{B}(\lambda,0) = +\hat{z}\frac{E_0}{c}}$



$\boxed{t = T/4}$: $\omega = \frac{2\pi}{T}$ $\quad \omega t = \frac{2\pi}{T}\left(\frac{T}{4}\right) = \frac{\pi}{2}$ $\quad \sin(\theta + \frac{\pi}{2}) = \cos\theta$ $\quad \cos(\theta+\frac{\pi}{2}) = -\sin\theta$

$\vec{E}(x,T/4) = \hat{y}E_0\cos(-kx+\frac{\pi}{2}) - \hat{z}E_0\sin(-kx+\frac{\pi}{2})$

$= \hat{y}E_0(-\sin(-kx)) - \hat{z}E_0\cos(-kx)$

$= \hat{y}E_0\sin kx - \hat{z}E_0\cos kx$

$\underline{\vec{E}(0,T/4) = -\hat{z}E_0}$ $\quad \underline{\vec{E}(\frac{\lambda}{4},\frac{T}{4}) = \hat{y}E_0}$ $\quad \underline{\vec{E}(\frac{\lambda}{2},\frac{T}{4}) = \hat{z}E_0}$

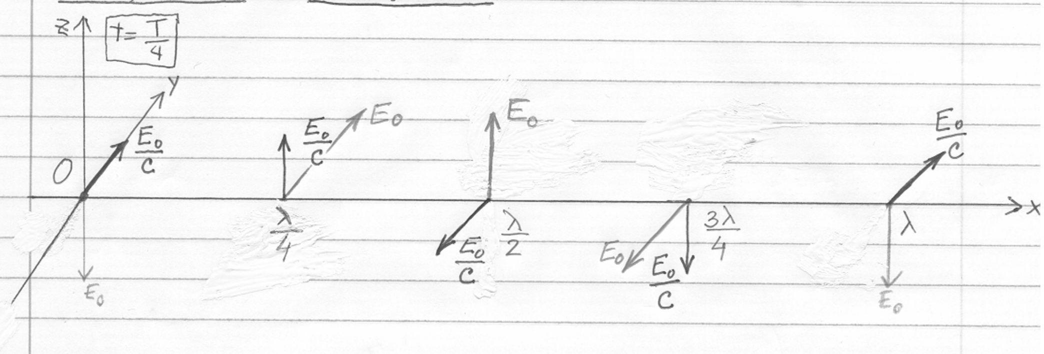
$\underline{\vec{E}(\frac{3\lambda}{4},T/4) = -\hat{y}E_0}$ $\quad \underline{\vec{E}(\lambda,T/4) = -\hat{z}E_0}$

$$\vec{B}(x, T/4) = \hat{y}\, E_0 \tfrac{1}{c} \sin(-kx + \tfrac{\pi}{2}) + \hat{z}\, \tfrac{E_0}{c} \cos(-kx + \tfrac{\pi}{2})$$

$$= \hat{y}\, \tfrac{E_0}{c} \cos kx - \hat{z}\, \tfrac{E_0}{c} \sin kx$$

$\underline{\vec{B}(0, T/4) = \hat{y}\, \frac{E_0}{c}}$ $\quad$ $\underline{\vec{B}(\frac{\lambda}{4}, \frac{T}{4}) = +\hat{z}\, \frac{E_0}{c}}$ $\quad$ $\underline{\vec{B}(\frac{\lambda}{2}, \frac{T}{4}) = -\hat{y}\, \frac{E_0}{c}}$

$\underline{\vec{B}(\frac{3\lambda}{4}, \frac{T}{4}) = -\hat{z}\, \frac{E_0}{c}}$ $\quad$ $\underline{\vec{B}(\lambda, \frac{T}{4}) = \hat{y}\, \frac{E_0}{c}}$



This type of E $ M wave is called "circularly polarized" because both the $\vec{E}$ and the $\vec{B}$ fields spiral in a circle around the x-axis.

(5) $y(x,t) = f(x-vt) + g(x+vt)$

find $y(x,t)$ for $f(u) = g(u) = A\cos(ku)$

$$A\cos(ku) = Re[Ae^{iku}]$$

$$y(x,t) = Re[Ae^{ik(x-vt)}] + Re[Ae^{ik(x+vt)}]$$
$$= Re[Ae^{ikx}(e^{-ikvt} + e^{ikvt})]$$

Use: $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$$y(x,t) = Re[Ae^{ikx}(2\cos(kvt))]$$
$$= A(2\cos(kvt))\,Re[e^{ikx}]$$
$$= 2A\cos(kvt)[\cos kx]$$

$$\underline{y(x,t) = 2A\cos(kvt)\cos(kx)}$$

This gives us a standing wave.

Problem Set 5 Solutions

6 (a) $v_y(x,0) = 0 \qquad y(x,t) = f(x-vt) + g(x+vt)$

$t=0: \quad y(x) = f(x) + g(x)$

$$0 = v_y(x) = -v f'(x) + v g'(x)$$

$$f'(x) = g'(x)$$

$$f(x) = g(x) + \cancel{c}^{\,0}$$ because $y(x)$ must equal zero at large and small x

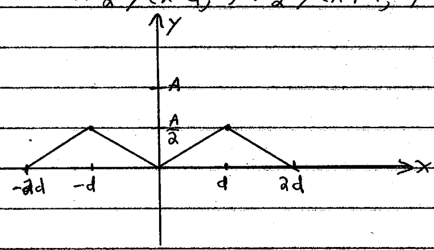
$$\boxed{f(u) = g(u)}$$

(b) $y(x,0) = f(x) + g(x) = 2f(x)$

$$f(x) = g(x) = \tfrac{1}{2} y(x,0)$$

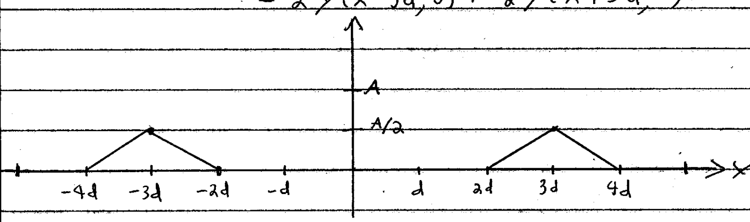
$t = d/v: \quad y(x,t) = f(x-d) + g(x+d)$

$$= \tfrac{1}{2} y(x-d,0) + \tfrac{1}{2} y(x+d,0)$$



$t = 3d/v: \quad y(x,t) = f(x-3d) + g(x+3d)$

$$= \tfrac{1}{2} y(x-3d,0) + \tfrac{1}{2} y(x+3d,0)$$



(c) $y(x,t) = f(x-vt) + g(x+vt)$

$y(x,0) = 0 = f(x) + g(x) \Rightarrow f(x) = -g(x)$

$$\boxed{f(u) = -g(u)}$$

Problem Set 5 Solutions

(d) $v_y(x,t) = -v f'(x-vt) + v g'(x+vt)$

$v_y(x,0) = -v f'(x) + v g'(x)$

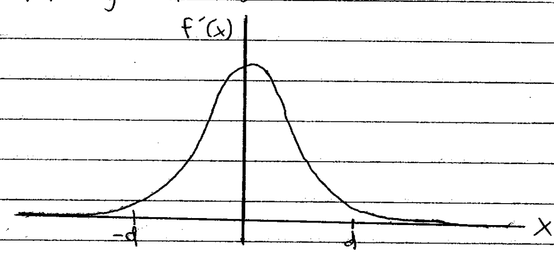
from (c), $-g(x) = f(x)$

$-g'(x) = f'(x)$

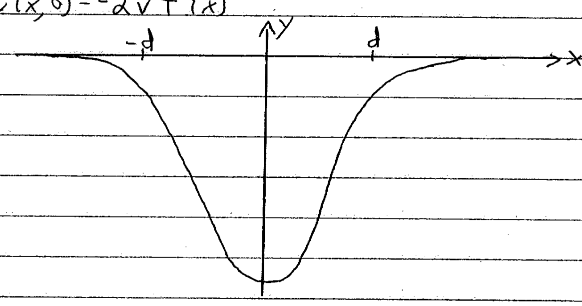
$v_y(x,0) = -v f'(x) - v f'(x)$

$v_y(x,0) = -2 v f'(x)$

Based on given graph for $f(u)$, we can graph $f'(u)$ (in our case substituting x for u)



$v_y(x,0) = -2 v f'(x)$



Problem Set 5 Solutions

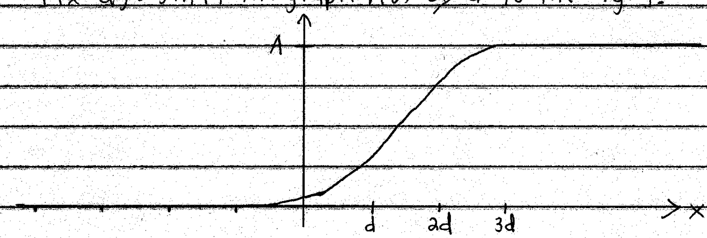
e) $y(x,t) = f(x-vt) + g(x+vt)$

$\quad = f(x-vt) - f(x+vt)$

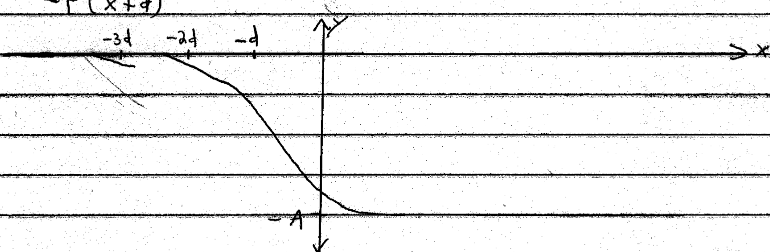
$y(x,\frac{d}{v}) = f(x-d) - f(x+d)$

$t = \frac{d}{v}$: $y = f(x-d) - f(x+d)$

$f(x-d)$: Shift the graph $f(u)$ by d to the right.

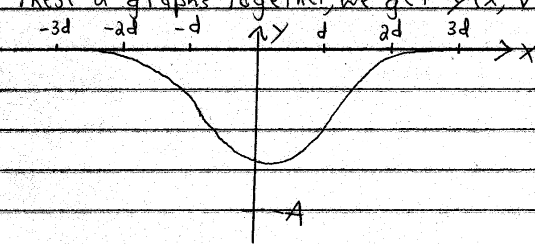


$-f(x+d)$



Adding these 2 graphs together, we get $y(x,\frac{d}{v}) = f(x-d) + g(x+d)$

$\boxed{t = \frac{d}{v}}$



$\boxed{t = \frac{3d}{v}}$