Physics 214 **Fall 2001**

Problem Set # 6

Reading for coming lectures:
Lecture Notes:(posted on the course web page under "Lectures")
Wave Phenomena II: Interference—Patterns from a single slit of infinitesimal width; Multiple slits.
Also: *Young & Freedman*, Sections 20.6, 37.1—37.3.

Problems: (due 10/23)

Skills to be mastered:

- *Superposition of pulses—understand superposition of both displacements and velocities;*
- *Be able to use the general solution $f(x - ct) + g(x + ct)$ of the wave equation to find particular solutions;*
- *Given a variety of initial conditions, be able to find the resulting time history (snapshots at later times, particle histories, velocity distributions);*
- *Be able to get time histories, snap shots, velocity distributions, etc. before, during and after reflection of pulses from fixed, free, and other boundary conditions;*
- *Be able to get time histories, snap shots, velocity distributions, etc. before, during and after reflection of pulses from changes in medium.*

1. Problem 1:
The velocity of a transverse wave on an infinitely long string is c. At $t = 0$ the string is flat (i.e., $y(x, t = 0) \equiv 0$) and has velocity distribution $v_y(x, t = 0) = V_0 \cos(kx + \pi/3)$ where V_0 and k are constants.

(a) Find the particular solution $y(x, t)$ to the wave equation for the string consistent with the above initial conditions.

(b) Does the solution found in (a) represent a *traveling wave*, a *standing wave*, or *neither*? Explain. If a traveling wave, in what direction is it moving? If a standing wave, re-write your solution for part (a) in the form $y(x, t) = A(x) \sin(\omega t)$ and find explicitly the function $A(x)$ in terms of no other quantities than V_0, c, and k.

2. Problem 2:
The velocity of a transverse wave on an infinitely long string is c. At $t = 0$ the string has shape given by $y(x, t = 0) = Ae^{-(x/a)^2}$ and velocity distribution $v_y(x, t = 0) = \dfrac{-2xcA}{a^2} e^{-(x/a)^2}$ where A and a are constants.

(a) Find the particular solution $y(x, t)$ to the wave equation for the string consistent with the above initial conditions.

(b) Does the solution found in (a) represent a *traveling wave*, a *standing wave*, or *neither*? Explain. If a traveling wave, in what direction is it moving?

3. Problem 3:
A pulse (shown on Figure 1) is heading in the x direction at 100 m/s towards the fixed end of a string (at $x = 0$).

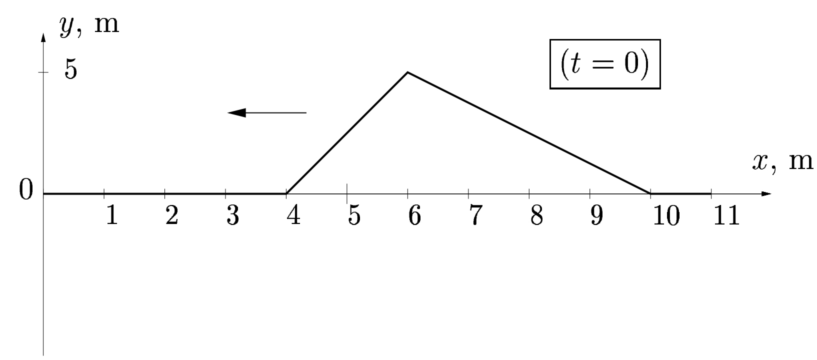

Figure 1: Travelling pulse.

(a) Sketch snapshots at $t = 0.05$ s and $t = 0.06$ s.

(b) Sketch the velocity distributions at $t = 0$, $t = 0.05$ s, and $t = 0.06$ s.

(c) Sketch a particle history for the endpoint ($x = 0$).

(d), (e), (f): Repeat parts (a), (b), and (c) if the end at $x = 0$ is free (in the y direction) instead of fixed.

4. Problem 4:
The function $f(u) = (\sin u)e^{-u^2}$ is sketched on Figure 2.

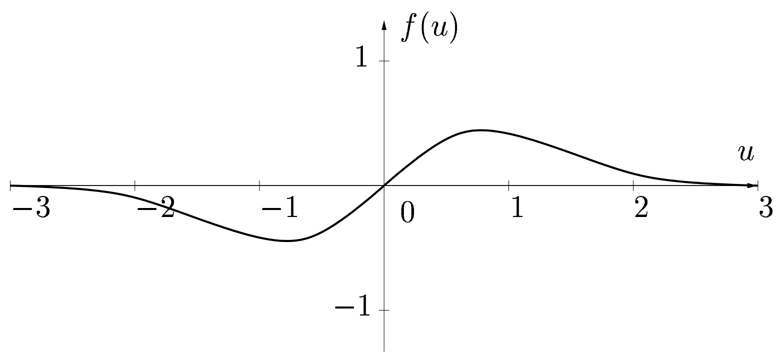

Figure 2: Sketch of the function $f(u) = (\sin u)e^{-u^2}$.

For each of the parts (a) through (d), describe briefly what has happened to the function (compared to $f(u)$) using the following terms:

- translated (shifted)—left, right, up, down, how far;
- reflected—left-right, up-down;
- scaled—contracted, expanded, in what direction (left-right or up-down), by what factor.

Note: Use a plotting program if you wish or sketch by hand. You don't need to hand in the plots or sketches.

(a) $f(u+2)$;

(b) $f(3u)$;

(c) $f(-u/2)$;

(d) $f(3u-2)$

5. Problem 5:
An industrious student decides to experiment with the effects of *drag* at a boundary using the setup shown on Figure 3. The string has linear mass density μ and is under tension τ. The left boundary has a very light (you may take it to be massless) ring that is free to move vertically except that it is attached to a dash-pot which applies a viscous drag force $F_y = -bv_y$ to the ring (see Figure 3).

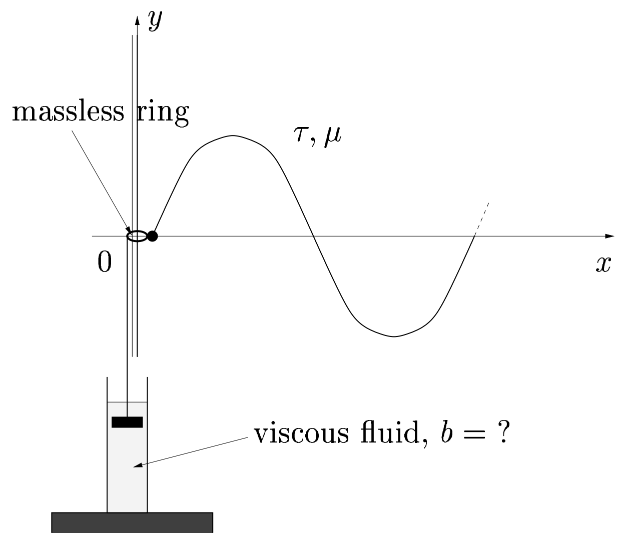

Figure 3: Boundary condition with drag.

(a) Derive the *boundary condition* at $(x = 0)$ in terms of the degree of freedom $y(x,t)$ and its derivatives evaluated at $x = 0$, and the constants given in the problem.

(b) By experimenting with different fluids with different 'drag coefficients' (b), she finds—amazingly—that she can send a pulse down the string without a reflection coming back! Find the value(s) of b for which no reflection occurs. Express your answer in terms of μ and τ.

Note: Proper termination of waveguides through *impedance matching* like this to avoid unwanted reflections is very important in many electronic communications applications.