Problem Set # 7

Reading for coming lectures: *Young & Freedman*, Sections 38.1—38-6.

Problems: (due 10/30) (Note: Although this problem set may appear long, it is mainly due to the presence of pictures and detailed descriptions. The actual tasks to be performed are not that lengthy.)

Skills to be mastered:

- *Be able to analyze interference of waves in the case of multiple point sources, such as N narrow slits, N radio-transmitters, etc.;*
- *Be able to use phasors to illustrate interference.*

1. Problem 1:
Young & Freedman use phasors to find the intensity in interference and diffraction experiments. A **phasor** is a **vector** that is **analogous to a complex number.** Adding complex numbers in the complex plane is just like adding vectors in the xy-plane. To add vectors, you sum the x-components to find the x-component of the sum and you sum the y-components to find the y-component of the sum. To add complex numbers, you sum the real parts to find the real part of the sum and you sum the imaginary parts to find the imaginary part of the sum. Thus the x**-component** of a phasor represents the **real** ($\Re\mathfrak{e}$) part of a complex number and the y**-component** represents the **imaginary** ($\Im\mathfrak{m}$) part. The **angle** of the phasor with the x-axis represents the **phase** of the complex number; the **length** represents the **magnitude.**

For instance, the sum $2 + 4e^{i\pi/3} \approx 5.3e^{i0.714\pi}$ can be represented by the phasor diagram below. We can use

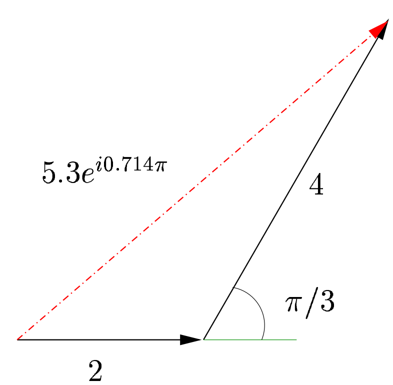

Figure 1: Adding complex numbers with phasors.

phasors to illustrate the sum inside the absolute value bars in Eq. (8) of the Lecture Notes. Each phasor represents the complex amplitude of the light coming from one source. When we sum these phasors, the resultant phasor represents the complex amplitude of the superposition of light coming from all of the sources.

Draw phasor diagrams to illustrate the ***following cases of interference. Label magnitudes and angles.*** If there is more than one possibility, illustrate ***all*** the possibilities.

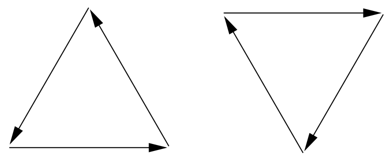

Figure 2: Phasor diagrams for 3-slit interference.

Example: Minimum for 3 thin slits. Answer:

Now you try it!

(a) Principal maximum for 4 thin slits:

(b) Minimum for 4 thin slits:

(c) Secondary maximum for 4 thin slits (don't worry about making the angles exact—an approximation is fine):

(d) Maximum for two thin slits where one is twice as wide as the other ($I_2 = 2I_1$):

2. Problem 2:

The engineering students at an unnamed Ivy League institution were operating a pirate radio station which broadcasted the problem set solutions each week from a transmitter somewhere in the Engineering quad. Figure 3 shows a map of the quad. The square boxes represent the buildings.

To locate the transmitter, the faculty set up a detector in the center of the quad (indicated by a small circle in the figure), where they measured the direction of the electric and magnetic fields of the transmitted wave. They found that the electric field had maximum magnitude 1 V/m and that when the magnetic field pointed up, the electric field pointed to the northeast, as Figure 3 shows.

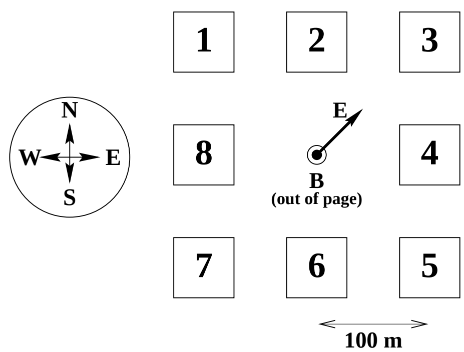

Figure 3: Pirate radio transmission.

(a) **Transmitter location:** Assuming that the radio transmission consists of a single plane wave of the type we discussed in class, ***what is the number*** of the building containing the transmitter?

(b) **Confusing the faculty:** Anticipating the faculty's technique, the students had actually arranged two transmitters, one in Building 2 and one in Building 8, to produce the field directions shown in Figure 3. ***Sketch*** on the diagram below, *using the same conventions as in Figure 3*, the directions of ***both*** the electric ***and*** magnetic fields coming from ***each*** transmitter.

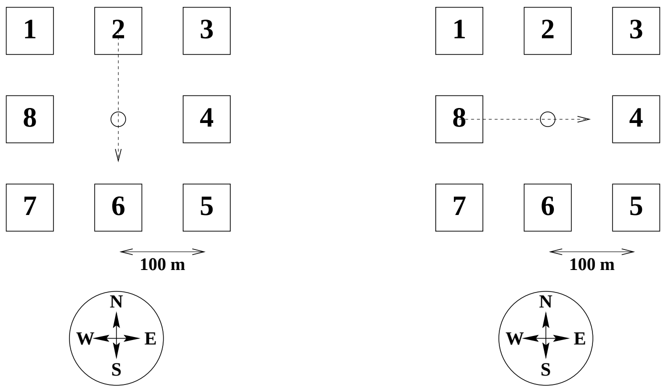

Figure 4: Confusing the faculty.

(c) **Trick discovered:** In the case of the two transmitters, the faculty eventually discovered the students' trick when they measured the magnitude of both the electric field *and* the magnetic field. ***Explain briefly*** (one or two sentences), how the faculty could know that there was more than one transmitter.

3. Problem 3 (Interference in sound):
Two small speakers (A and B) are set up at distance d apart from each other, each at a distance $R >> d$ from the wall of a large, curved dormitory building. (See Figure 5.) Both speakers emit sound equally in all directions, with the same frequency f and same phase, but their intensities are different. The intensities of the sound arriving at the wall from A and B alone are $I_A = 16\mathrm{W/m}^2$ and $I_B = 25\mathrm{W/m}^2$, respectively. Finally, the speed of sound is c.

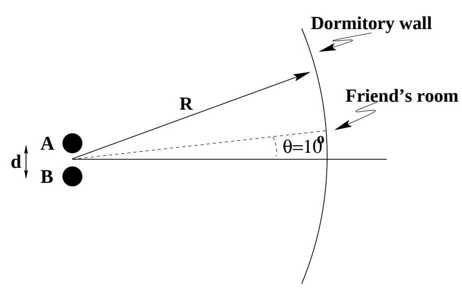

Figure 5: Interfering speakers.

(a) **Propagation number:** In terms of f, c and any relevant mathematical constants (e, π etc.), ***what is the propagation number*** k for the waves emitted by the speakers?

(b) **Maxima:** In terms of d, k and any relevant mathematical constants, ***at what angles*** θ will the sound intensity be ***maximum?*** *And*, ***what is the intensity*** we should expect at these ***maxima?***

Hint: *You can do this without part (d), but if you can do that part, you may wish to do it first.*

(c) **Minima:** In terms of d, k and any relevant mathematical constants, ***at what angles*** θ will the sound intensity be ***minimum?*** *And*, ***what is the intensity*** we should expect at these ***minima?***

Hint: *You can do this without part (d), but if you can do that part, you may wish to do it first.*

(d) **Full formula for intensity at the wall:** In terms of d, θ, k, and any relevant mathematical constants, ***write a expression for the intensity*** of the sound which will be observed at angle θ at the wall. ***Simplify your expression*** so that it involves *no complex numbers.*

(e) **Moving the maxima:** Suppose that $d = 2$m and $k = 5\text{m}^{-1}$, and one of your "friends" lives in a room located at $\theta = 10^o$ along the wall. ***What minimum phase shift*** would you need between the two speakers so that your "friend" receives the ***maximum possible sound intensity?*** For this to happen, should the phase of A ***lead*** or ***lag*** the phase of B?

Note: If the waves coming from A hit their maximum **before** the waves from B do, then we say that A "leads" B. If the waves from A hit their maximum **after** the waves from B do, then we say that A "lags" B.

4. Problem 4:

Five radio towers located $d = 300$ m apart from one another transmit radio waves of wave-length $\lambda = 100$ m. Each tower transmits the same power in every horizontal direction. The electric currents in the towers which produce the radio waves have **different phases:** $\phi_1 = 0$, $\phi_2 = \pi/2$, $\phi_3 = \pi$, $\phi_4 = 3\pi/2$, and $\phi_5 = 2\pi$. (See Figure 6.)

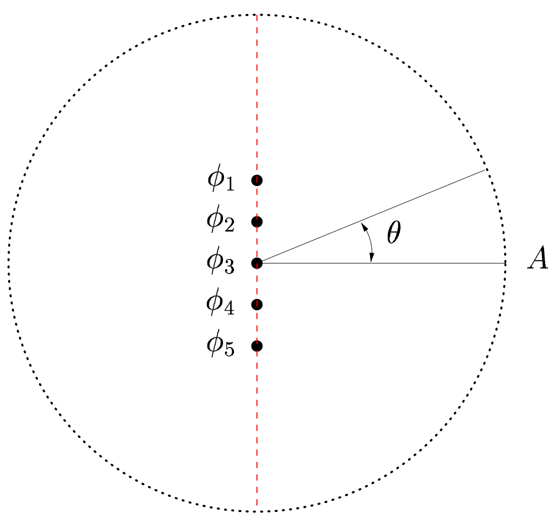

Figure 6: Interfering radio towers.

(a) **Central maximum or central minimum:** ***What is the difference in the path lengths*** traveled by the waves which arrive at point A ($\theta = 0$). Is this an interference ***maximum*** or a ***minimum***?

(b) **First maximum:** ***Find the first nonzero angle*** θ_1 of an ***intensity principal maximum*** in the first quadrant $0 < \theta_1 \leq \pi/2$

(c) **Higher-order maxima:** ***Derive an expression*** that gives the locations of ***all*** the principal maxima in the first quadrant: $0 < \theta_1,\ \theta_2,\ \ldots,\ \theta_n \leq \pi/2$.