Problem Set # 9

Reading on current material and for coming lectures: *Young & Freedman*, Sections 40.1—40.3, 40.10, 41.1—41.4.

Problems: (due 11/29)

Skills to be mastered:

- *Be able to determine energy densities (kinetic, potential, total) given the solution* $y(x,t)$.
- *Be able to determine the total energy (i.e., integrated density) given the solution* $y(x,t)$.

1. Problem 1 (Total Energy in a Traveling Wave):
Four identical strings, each with equal tensions $\tau = 9$ N and mass per unit length $\mu = 1$ kg/m, carry traveling wave pulses, as shown on Figure 1, and traveling to the right. (Figure 1 represents a snapshot of the strings at $t = 0$.)

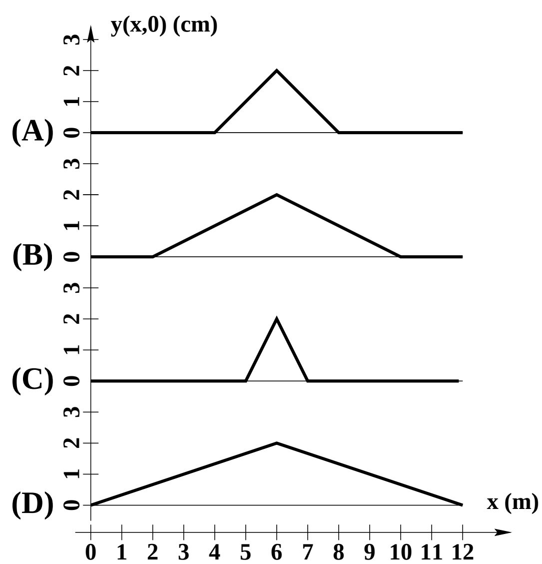

Figure 1: Total energy of various pulses.

(a) ***Calculate*** the ***kinetic energy density*** $ke(x)$ and the ***potential energy density*** $pe(x)$ for each of the pulses. ***Is it true*** that $ke(x) = pe(x)$?

(b) ***Calculate the energy density*** $e(x)$ for ***each*** of these pulses. ***Which one*** (A, B, C, or D) has the ***lowest energy density in the region of the pulse?***

(c) ***Calculate the total energy*** for ***each*** of these pulses. Which one (A, B, C, or D) has the ***highest*** total energy?

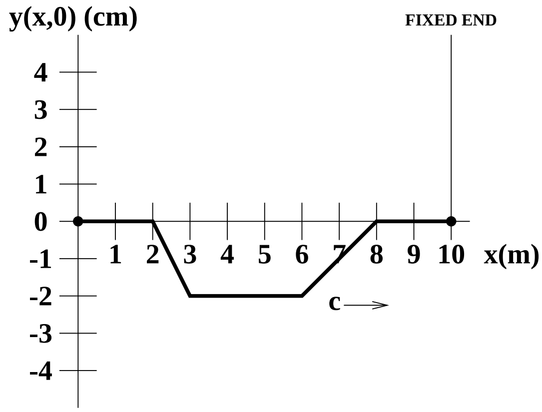

Figure 2: Power in a travelling pulse.

2. Problem 2 (Power in a Traveling Wave):
Figure 2 above shows a traveling wave propagating on a string at time $t = 0$. The wave is propagating to the right (positive x-direction). The tension in the string is $\tau = 9$ N, and the string has a mass per unit length of $\mu = 1$ kg/m. The string has a length of 10 m, and has a *fixed end* at $x = 10$ m.

(a) ***Draw a graph*** of the ***transverse velocity*** (chunk speed) of the wave at $t = 0$, as a function of x. Label your axes clearly and be sure to give units.

(b) ***Draw a graph*** of the ***power*** in the wave at t=0, as a function of x. Label your axes clearly and be sure to give units.

(c) ***Draw a graph*** of the ***amplitude*** of the wave at $t = 1$ s, as a function of x. Label your axes clearly and be sure to give units.

(d) ***Draw a graph*** of the ***transverse velocity*** (chunk speed) of the wave at $t = 1$ s, as a function of x. Label your axes clearly and be sure to give units.

(e) ***Draw a graph*** of the ***power*** in the wave at $t = 1$ s, as a function of x. Label your axes clearly and be sure to give units.

Note: In **Problems 3**, **4**, and **5**, **"The Pulse"** refers to the pulse shown on Figure 3 (a snapshot at $t = 0$). In each problem, The Pulse is initially moving down the string to the left at wave speed c. The string *does not necessarily* end at $x = 0$. Everywhere in these two problems, **"plot"** means: use graph paper, plot carefully, and label numerical values on the axes.

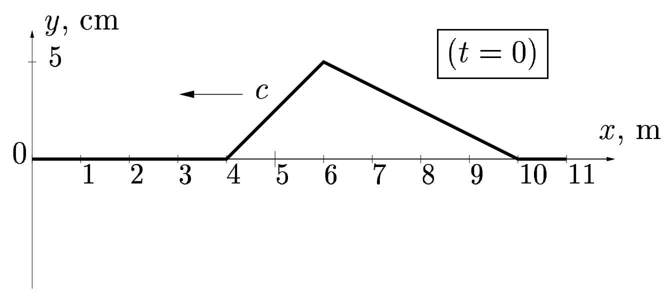

Figure 3: The Pulse.

3. Problem 3:
The Pulse is headed at speed $c = 100$ m/s toward a change in medium at $x = 0$. The string does not end at x = 0; rather, it is tied to a more massive string at $x = 0$ such that $\mu' = 9\mu$ (where μ is the linear mass density for $x > 0$ and μ' is the linear mass density for $x < 0$).

<u>Hint:</u> Use your results from *Problem Set #8, Problem 3*, replacing $B \to \tau$ and $\rho \to \mu$.

(a) ***Plot the snapshots*** for $t = 0.06$ s and 0.12 s;

  <u>Hint:</u> Draw the 0.12 s snapshot first.

(b) ***Find the total energies*** of the ***incoming, reflected and transmitted*** pulses. ***Verify that energy is conserved.***

4. Problem 4:
The Pulse moves on a string with $\mu = 0.001$ kg/m and $\tau = 10$ N. The string ends at $x = 0$.

(a) At $t = 0$, ***plot*** the ***kinetic energy density*** $ke(x)$, the ***potential energy density*** $pe(x)$, the total energy density $e(x)$, and the ***instantaneous power*** $P(x)$;

(b) How does $e(x)$ ***compare*** with $P(x)$?

  <u>Hint:</u> How fast does energy move along the string?

(c) ***What is the power at*** $x = 0$ ***as a function of time*** if the string has a ***fixed end*** there? ***Explain your result in terms of energy conservation.***

(d) ***Repeat*** (c) if $x = 0$ is a ***free end***.

5. Problem 5:
In *Problem 5* of *Problem Set #6*, you found that if $b = \tau/c = \sqrt{\tau\mu}$, there is no reflected pulse. Suppose The Pulse is moving at $c = 100$ m/s toward $x = 0$, at which point this "perfect absorber" has been set up; in other words the string ends at $x = 0$ and the endpoint is attached to a dashpot with just the right value of b to completely absorb The Pulse.

(a) ***Plot*** $P(t)$, the ***power as a function of time***, at the ***endpoint*** $x = 0$.

(b) The rate at which energy is dissipated by the dashpot is $P_d = F_y v_y$, where F_y is the viscous drag force and v_y the y-velocity of the endpoint (i.e., of the piston). ***Is*** $P(t) = P_d(t)$***? Explain briefly***, based on ***energy conservation***.