CORNELL UNIVERSITY

Department of Physics

Physics 214 Prelim II Fall 2003

NAME: Answers

SECTION:

Instructions

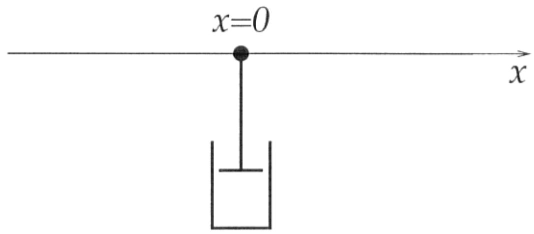
— Closed book; no notes. You may use a calculator.
— Check that you have all **14** pages (including cover page). The formula sheet is distributed separately.

Problem	Score	Grader
1. (25 pts)		
2. (25 pts)		
3. (25 pts)		
4. (25 pts)		
Total (100 pts)		

Contents

1 Problem 1: String with a Shock Absorber [25 points]

A sinusoidal wave of complex amplitude $\underline{A}$ is sent from left to right along a very long stretched string which is oriented in the x-direction. The speed of transverse waves all along this string is c, the tension is τ. At $x = 0$ a shock aborber is attached to the string (see figure). The force exerted by the shock absorber on the string is given by $\vec{F} = -b\vec{v}$, where $\vec{v}$ is the transverse velocity of the string at the point of contact. The point of the string where the shock absorber is attached is infinitesimal and therefore has zero mass.



Solutions to the wave equation for the string are given by

$$\begin{aligned} y_0(x \le 0, t) &= \Re\left[\underline{A}e^{-i\omega t}(e^{ikx} + \underline{R}e^{-ikx})\right] \\ y_1(x \ge 0, t) &= \Re\left[\underline{A}e^{-i\omega t}\underline{T}e^{ikx}\right] \end{aligned}$$

(a) (7 points)

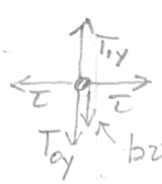
Explain why the following two equations hold at point $x = 0$:

(1) $$y_0(x=0,t) = y_1(x=0,t),$$

(2) $$\tau\left(\frac{\partial y_1}{\partial x}\bigg|_{x=0} - \frac{\partial y_0}{\partial x}\bigg|_{x=0}\right) - b\frac{\partial y_1}{\partial t}\bigg|_{x=0} = 0.$$

(1) : continuity at $x = 0$ (consistency)

(2) net force on point at $x=0$ is zero (equilibrium)



$$T_{1y} - T_{0y} - bv = 0$$

$$\tau\frac{\partial y_1}{\partial x}\bigg|_{x=0} - \tau\frac{\partial y_0}{\partial x}\bigg|_{x=0} - b\frac{\partial y_1}{\partial t}\bigg|_{x=0} = 0 \qquad v = \frac{\partial y_1}{\partial t}\bigg|_{x=0} = \frac{\partial y_0}{\partial t}\bigg|_{x=0}$$

(b) (10 points)

Find $\underline{R}$ and $\underline{T}$ in terms of b, c, τ, and any numerical constants.

From Equ. (1) $\quad \underline{A}e^{-i\omega t}(1+\underline{R}) = \underline{A}e^{-i\omega t}\underline{T} \qquad \underline{R} = \underline{T} - 1$

From (2) $\quad \tau ik\underline{T}\underbrace{e^{ik\cdot 0}}_{1} - \tau(ik - ik\underline{R}) - b(-i\omega)\underline{T} = 0$

leaving out $\underline{A}e^{i\omega t}$ on both sides

$$\tau k\underline{T} - \tau k + \tau k\underline{R} + b\omega\underline{T} = 0$$

$\omega = kc$

$$\tau\underline{T} - \tau + \tau\underline{R} + bc\underline{T} = 0$$

$$(\tau + bc)\underline{T} - \tau + \tau(\underline{T} - 1) = 0$$

$$(2\tau + bc)\underline{T} = 2\tau$$

$$\underline{T} = T = \frac{2\tau}{2\tau + bc} \quad \text{it is real}$$

$$\underline{R} = T - 1 = \frac{2\tau - (2\tau + bc)}{2\tau + bc} = \frac{-bc}{2\tau + bc}$$

(c) (8 points)

What are $\underline{R}$ and $\underline{T}$ in the limit of $b \to 0$ and $b \to \infty$? Explain each limit using physical principles, in a short phrase. (Even if you did not complete part (b), take a good guess!)

$b \to 0$ plain string w/o discontinuity $R = 0 \quad T = 1$

$b \to \infty$ string is fixed at $x = 0 \quad T \to 0$

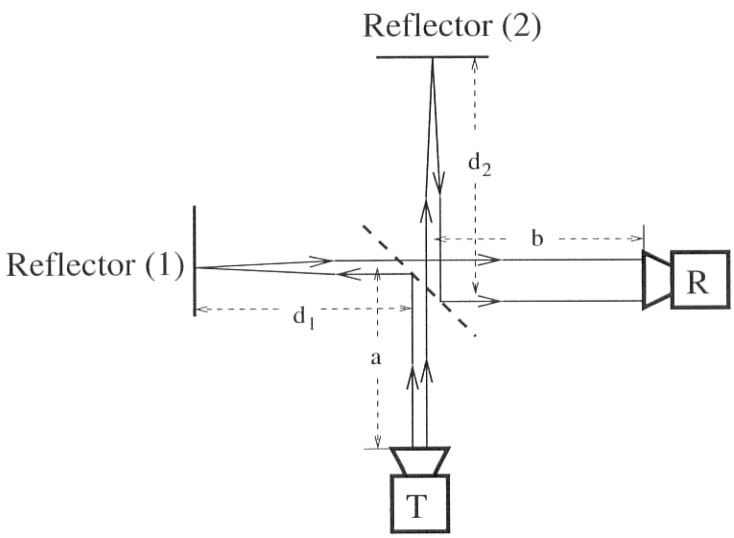
$$R = \frac{-c}{\frac{2\tau}{b} + c} \longrightarrow -1$$

2 Problem 2: Michelson Interferometer with Microwaves [25 points]

Recall the microwave lab, in particular the experiments with a Michelson interferometer (see figure). (T) is the transmitter and (R) is the receiver. Full reflectors (1) and (2) are at the ends of two arms of the interferometer, a partial reflector is at the center.

Two waves contribute to the final intensity. The first travels from the transmitter, is reflected by the partial reflector toward full reflector (1), then travels back through the partial reflector to the receiver. This wave travels a total path length of $a + 2d_1 + b$. (See figure.) The second travels from the transmitter, through the partial reflector toward full reflector (2), then travels back to the partial reflector, where it is reflected toward the receiver. This wave travels a total path length of $a + 2d_2 + b$.

Reflector (1) is being moved in the course of the experiment.



(a) (5 points)

In terms of d_1, d_2, and the wave vector k, what is the phase difference between the two waves arriving at the receiver?

Path difference is $(d_2 - d_1) \times 2$

Phase difference $\Delta\phi = 2k(d_2 - d_1)$

(b) (6 points)

As you move Reflector (1), the receiver registers maxima of intensity at many reflector positions d_1 where the two contributing waves interfere constructively. You measure five of these positions to be at $d_1 = 20.0$ cm, 21.9 cm, 24.1 cm, 26 cm, and 28 cm. ***What (approximately) is the wavelength of the microwaves?*** Two significant figures are sufficient.

1,9
2,2
1,9
2.0

$\Delta d_1 = (2.0 \pm 0.07)$ cm

Because of the reflection, beam travels an extra distance $2 \cdot \Delta d_1$ when Reflector (1) is moved.

$\lambda = 4.0$ cm

(c) (7 points)

Suppose you have your transmitter set at the same frequency as in part **(b)**, and that you set $d_1 = d_2$ so that the phase difference between the two waves arriving at the receiver is zero. You now insert a 1.0 cm thick slab of polyethylene with index of refraction $n = 1.6$ between the central partial reflector and Reflector (1). You leave the receiver as well as Reflector (2) in place. Note that, while the frequency is unchanged inside the slab, the speed of light in polyethylene is c/n, and the wave vector changes accordingly.
If you did not get a result for **(b)**, use $\lambda = 3.0$ cm (not the answer to **(b)**).

Calculate the phase difference $\Delta\phi$ between the two waves that has been introduced by inserting the polyethylene slab.

$\Delta\phi = 2\,(k_{Poly} - k_{air})\,d_{slab}$ $\quad d_{slab} = 1$ cm

because of reflection

$\Delta\phi = 2\,(n-1)\,k_{air}\,d_{slab}$

$k_{air} = \frac{2\pi}{4\,\text{cm}}$

$= 2\,(1.6-1)\,\frac{2\pi}{4\,\text{cm}}\,1\,\text{cm} = 0.6\,\pi$

$k_{Poly} = n\,k_{air}$

because $\omega = k_{air}\,c = k_{Poly}\,\frac{c}{n}$

$c_{Poly} = \frac{c}{n}$

(d) (7 points)

Calculate the smallest distance ΔX that you need to move Reflector (1) toward the center in order to reestablish zero phase difference.

To receive full credit, you must show how you arrive at your result. Merely inserting numbers in a formula that you remember from the lab is not sufficient.

$$\frac{2\Delta X}{\lambda} = \frac{\Delta\phi}{2\pi}$$ $2\Delta X$ because of reflection

$$2\Delta X = \frac{\lambda}{2\pi}\underbrace{2(n-1)\frac{2\pi}{\lambda}d_{slab}}_{\text{from (c)}} = 2(n-1)d_{slab}$$

$$\underline{\Delta X} = (n-1)d_{slab}$$
$$= \underline{0.6\,cm}$$

or: $k(2\Delta x) = \Delta\phi$

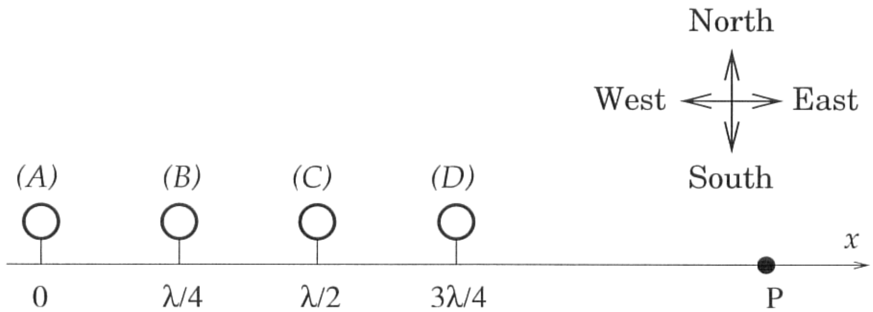
$$\frac{2\pi}{\lambda}2\Delta x = 0.6\pi$$

$$\frac{2\pi}{4\,cm}2\Delta x = 0.6\pi$$

$$\Delta x = 0.6\,cm$$

3 Problem 3: Four Radio Towers [25 points]

Four radio towers are emitting radiowaves with equal power. The wavelength is λ and the angular frequency is $\omega = 2\pi c/\lambda = kc$. The towers (A), (B), (C) and (D) are situated along the x-axis (East-West direction), at a distance of $\lambda/4$ from each other. (An array like this is on South Hill in Ithaca). Let Tower (A) be at $x = 0$, Tower (B) at $x = \lambda/4$ etc.



Interestingly, these radio transmitters oscillate out of phase with each other, such that the individual electric fields, at a distance d from any one of these towers, are given by

$$\begin{aligned}
E_A(d,t) &= \Re(\underline{A}f(d)e^{-i\omega t}e^{ikd}) \\
E_B(d,t) &= \Re(\underline{A}f(d)e^{-i\omega t}e^{i\pi/2}e^{ikd}) \\
E_C(d,t) &= \Re(\underline{A}f(d)e^{-i\omega t}e^{i\pi}e^{ikd}) \\
E_D(d,t) &= \Re(\underline{A}f(d)e^{-i\omega t}e^{i3\pi/2}e^{ikd})
\end{aligned}$$

where $f(d)$ describes the attenuation of radio signal with distance.

(a) (8 points)

Consider a point P at $x = R$ on the positive x-axis. The point is far away from the towers, so that $f(d) \approx f(R)$ is the same for all four waves. ***Fill in the missing phases in the following equations for the fields at*** $x = R$ ***(Note:*** $\exp[x] \equiv e^x$***):***

$$\underline{E}_A = \underline{A}f(R)e^{-i\omega t}e^{ikR}\exp\left[\ 0\ \right] \qquad 0$$

$k\lambda = 2\pi$

$$\underline{E}_B = \underline{A}f(R)e^{-i\omega t}e^{ikR}\exp\left[i\frac{\pi}{2} - ik\frac{\lambda}{4}\right] \qquad i\frac{\pi}{2} - ik\frac{\lambda}{4} = i\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 0$$

$$\underline{E}_C = \underline{A}f(R)e^{-i\omega t}e^{ikR}\exp\left[i\pi - ik\frac{\lambda}{2}\right] \qquad i\pi - ik\frac{\lambda}{2} = i(\pi - \pi) = 0$$

$$\underline{E}_D = \underline{A}f(R)e^{-i\omega t}e^{ikR}\exp\left[i\frac{3\pi}{2} - ik\frac{3\lambda}{4}\right] \qquad i\frac{3\pi}{2} - ik\frac{3\lambda}{4} = i\left(\frac{3}{2}\pi - \frac{3}{2}\pi\right) = 0$$

exp [0] for all factors

(b) (10 points)

Let I_0 denote the intensity that would be received at point P if only one tower were operating. ***What is the intensity I_{tot} received at P when all four towers are operating?*** (Simplify your result as much as possible. Express the final result as a real number times I_0.)

$$\frac{I_{tot}}{I_0} = \frac{|\underline{E}_A + \underline{E}_B + \underline{E}_C + \underline{E}_D|^2}{|\underline{E}_A|^2}$$

Since $\underline{E}_A = \underline{E}_B = \underline{E}_C = \underline{E}_D$ from (a)

$$\underline{I_{tot}} = \frac{|4\underline{E}_A|^2}{|\underline{E}_A|^2} I_0 = \underline{16 I_0}$$

(c) (7 points) *Challenge Problem!*

Using the same analysis as above (you need not write out each step), what is the ratio I_{tot}/I_0 at a distant point due west of the array of towers (on the negative x-axis)? Explain your reasoning.

E_A phase 0

E_B phase is $\frac{\pi}{2} + k\frac{\lambda}{4} = \pi$ $\Big\}$ destructive

E_C $\pi + k\frac{\lambda}{2} = 2\pi$

E_D $\frac{3}{2}\pi + k\frac{3\lambda}{4} = \left(\frac{3}{2}\pi + \frac{3}{2}\pi\right) = 3\pi$ $\Big\}$ destructive

combined electric field is zero

$I_{tot} = 0$

What is the ratio I_{tot}/I_0 at a distant point due north or south of the array of towers ("up" or "down" on the page)?

see p. 8

E_A phase 0

E_B phase $\pi/2$

E_C phase π

E_D phase $\frac{3}{2}\pi$

in far field
all distances the same
but phase difference between
emitted waves causes cancellations

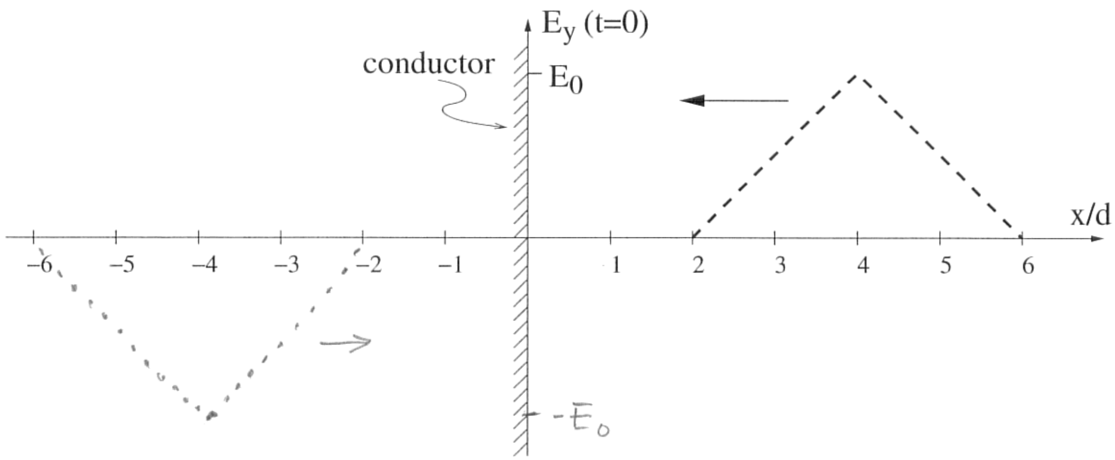
(A) and (C) interfere destructively

(B) and (D) " "

$I_{tot} = 0$

4 Problem 4: Reflection of an Electromagnetic Pulse [25 points]

An electromagnetic pulse (plane wave) is traveling with speed c in the negative x-direction towards an infinite conducting plate, which is placed in the yz-plane at $x = 0$. The electric field vector $\vec{E}(x,t)$ is polarized in the y-direction. At $t = 0$, the electric field of the pulse has the shape shown in the graph below.



Since the conductor represents a boundary, our pulse will be reflected at $x = 0$. To construct reflected pulses on the following pages, we imagine there is a right-moving pulse coming from the left. We use the following convention in sketching:

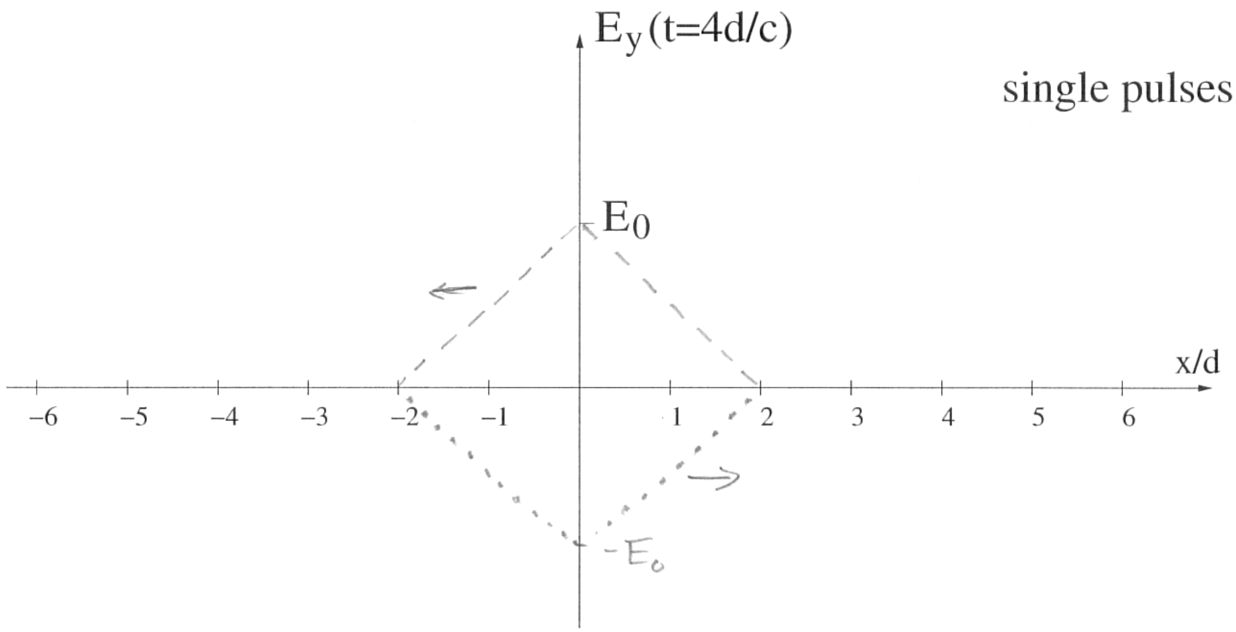
- left-traveling pulse – – – – –
- right-traveling pulse . . . .
- superposition of both ________________

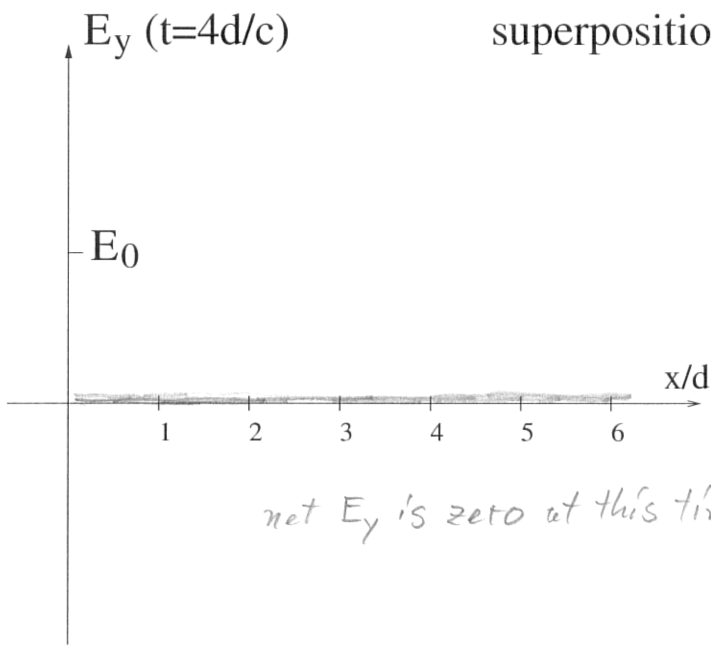
(a) (6 points)

The boundary condition associated with a perfect conductor is that $\vec{E}(x = 0, t) = 0$ ***for all times. Given this, sketch on the graph above the electric field associated with the imagined reflected pulse for time*** $t = 0$. (Remember to use the conventions above for left/right moving pulses!)

(b) (6 points)

Using the convention from the previous page, sketch the incoming and reflected electric field pulses at time $t = 4d/c$ on the upper graph, and sketch the net electric field in the physical region ($x > 0$) at the same time $t = 4d/c$ on the lower graph.

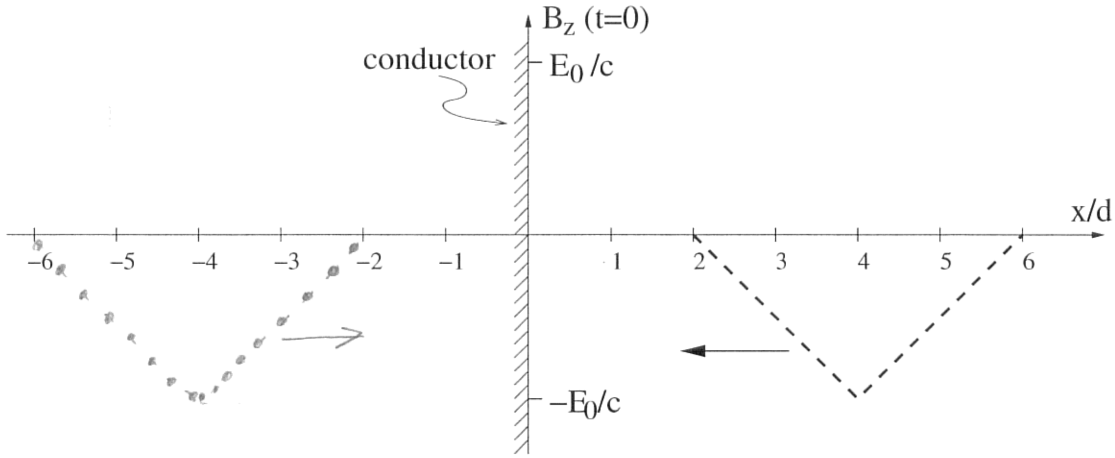




net E_y is zero at this time

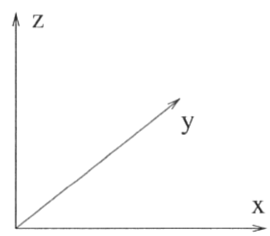
(c) (7 points)

The sketch below shows the associated magnetic pulse $\vec{B}(x,t)$ traveling to the left, at $t=0$.



On the figure above, sketch the (imagined) magnetic reflected pulse that is moving from left to right, at $t=0$.

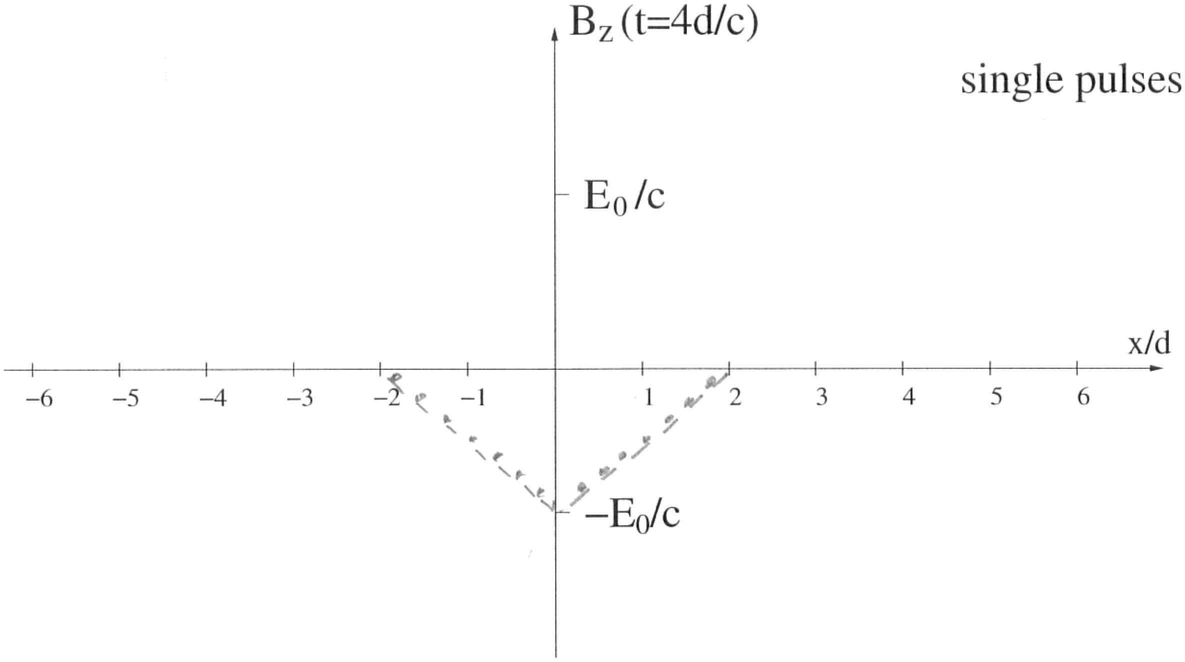
Hint: Pay attention to the relation between the directions of the electric field vector, the magnetic field vector, and the direction of propagation. The orientation of the coordinate system is shown below: the $+x$-axis is to the right, the $+z$-axis is up, and the $+y$-axis is "into the page".

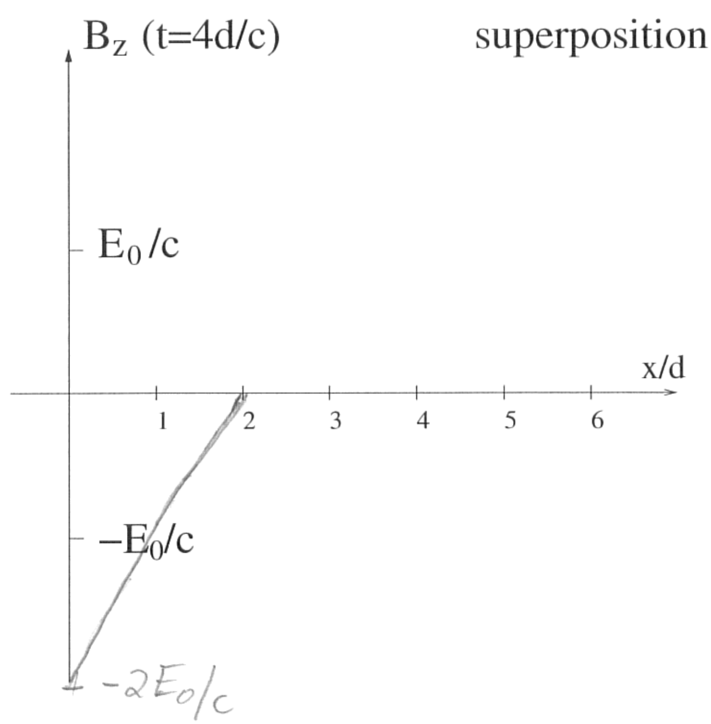


The reflected E_y-pulse is $-E_y$, pointing out of page
By right hand rule $B_{refl.}$ has to be down for an EM-field traveling to the right

(e) (6 points)

Using the same conventions as above for the electric field, sketch the incoming and reflected magnetic field pulses at time $t = 4d/c$ on the upper graph, and sketch the net magnetic field in the physical region ($x > 0$) at the same time $t = 4d/c$ on the lower graph.





END OF EXAM