Physics 214 **Fall 2003**

Exercise Problems for the Final Exam ("Prelim # 3")

Note: Below is a list of some exercise problems that could be used as a supplement to your preparation for the exam. These problems have been given as homework or exam problems in previous years. Note that the list is **not comprehensive** and it is **not meant to cover all the topics** taught in the course. For instance, you certainly can expect a sum over histories problem to be on the final. (See LN "Introduction to complex represenation for waves" for a worked out example.) You are **primarily responsible for the material presented in Lectures (everything from N narrow-slit diffraction on), Problem Sets 9—11, and Lab III.** You are expected to understand all the concepts in this material (including derivations) and to apply them creatively to different situations. Some **additional information** can be found on the course web page.

1 Off-Axis Diffraction

A single slit of width a is illuminated by an off-axis source. The light from the source has wavelength λ and hits the slit at an angle θ_0. Our goal in this problem is to find out how the single-slit diffraction pattern at a distance screen is changed by having the source off-axis (compared to the "regular" setup where $\theta_0 = 0$).

(a) Consider the slit to be N small slits spaced a distance $d = a/N$ apart, where N is very large. The waves emerging from the various slits do not start in phase, since the light from the source had to travel different distances to reach them. Assuming that the light arrives at the top slit ($n = 0$) with phase ϕ_0, find the phase of the light that arrives at the bottom slit ($n = N - 1$) in terms of θ_0, ϕ_0, a, and λ.

(b) Generalize your result from (a) to find the phase at the n^{th} slit ϕ_n. The n^{th} slit is at a distance $\frac{n}{N}a$ below the top slit. Write your answer in the form $\phi_n = \phi_0 + n\Delta\phi$. Express $\Delta\phi$ in terms of θ_0, ϕ_0, a, l, and N (as needed).

(c) Now use the ϕ_n to find the intensity $I(x)$ at the screen due to the N slits. [*Hint:* Follow the procedure used in section 3.2.1 of the Notes, but this time the ϕ's are not all equal.]

(d) Take the limit $N \to \infty$ to find the intensity at the screen due to the slit of width a. A correct result cannot have any n's or N's in it (why not?). Check your result for the special case $\theta_0 = 0$.

(e) Explain how the intensity pattern differs from the pattern in the "regular" setup. At what angle θ does the "central" maximum appear?

2 Diffraction at Different Wavelengths

Young & Freedman, Problem 38-6.

3 Multiple Finite Slits

The graph in Figure 1 shows the intensity (in arbitrary units) as a function of position (y) on a screen which is 1.00 m from an aperture illuminated at normal incidence by laser light of wavelength 0.628 μm. The aperture consists of some number of equally spaced, equally wide slits. Determine the number of slits N, the slit width a, and the center-to-center slit separation d.

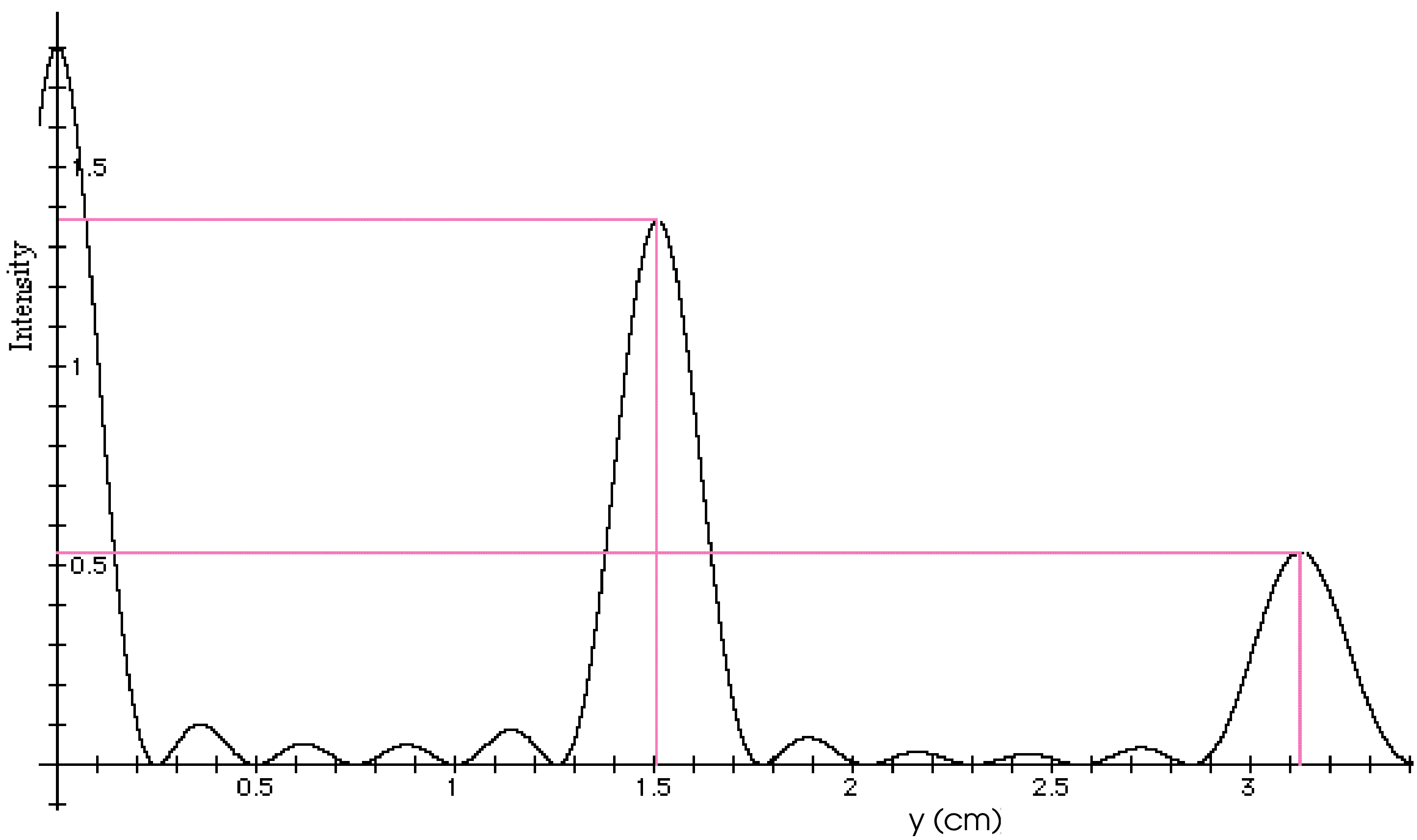

Figure 1: Intensity on the screen from multiple finite slits.

4 Energy in Pulses

In the following three problems, **"The Pulse"** refers to the pulse shown on Figure 2 (a snapshot at $t = 0$). In each problem, The Pulse is initially moving down the string to the left at wave speed c. The string *does not necessarily* end at $x = 0$. Be sure to plot carefully and label numerical values on the axes.

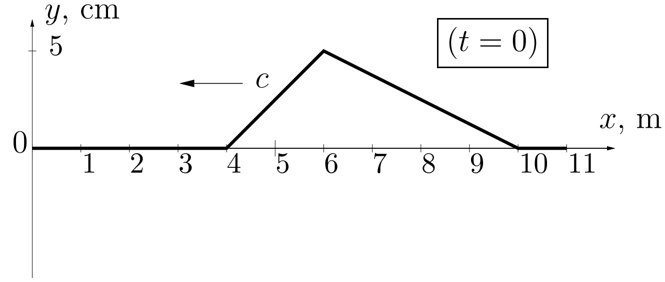

Figure 2: The Pulse.

4.1 Change in Medium

The Pulse is headed at speed $c = 100$ m/s toward a change in medium at $x = 0$. The string does not end at x = 0; rather, it is tied to a more massive string at $x = 0$ such that $\mu' = 9\mu$ (where μ is the linear mass density for $x > 0$ and μ' is the linear mass density for $x < 0$).

(a) ***Plot the snapshots*** for $t = 0.06$ s and 0.12 s;

Hint: Draw the 0.12 s snapshot first.

(b) ***Find the total energies*** of the ***incoming, reflected and transmitted*** pulses. ***Verify that energy is conserved.***

4.2 Reflection

The Pulse moves on a string with $\mu = 0.001$ kg/m and $\tau = 10$ N. The string ends at $x = 0$.

(a) At $t = 0$, ***plot*** the ***kinetic energy density*** $ke(x)$, the ***potential energy density*** $pe(x)$, the total energy density $e(x)$, and the ***instantaneous power*** $P(x)$;

(b) How does $e(x)$ ***compare*** with $P(x)$?

Hint: How fast does energy move along the string?

(c) ***What is the power at*** $x = 0$ ***as a function of time*** if the string has a ***fixed end*** there? ***Explain your result in terms of energy conservation.***

(d) ***Repeat*** (c) if $x = 0$ is a ***free end***.

4.3 Perfect Absorber

Suppose The Pulse is moving at $c = 100$ m/s toward $x = 0$, at which point this "perfect absorber" has been set up; in other words the string ends at $x = 0$ and the endpoint is attached to a dashpot with just the right value of b to completely absorb The Pulse. (This value can be computed in the same way for string as was done for sound in *Problem 5* of *Problem Set #7*. The answer is: $b = \tau/c = \sqrt{\tau\mu}$.)

(a) ***Plot*** $P(t)$, the ***power as a function of time***, at the ***endpoint*** $x = 0$.

(b) The rate at which energy is dissipated by the dashpot is $P_d = F_y v_y$, where F_y is the viscous drag force and v_y the y-velocity of the endpoint (i.e., of the piston). ***Is*** $P(t) = P_d(t)$***? Explain briefly***, based on ***energy conservation***.

5 Infinite Square Well Potential

A particle of mass m is confined to a inpenetrable box of length $2a$. The box can be modeled by a **square well potential** of **infinite** depth, as the one shown on Figure 3.

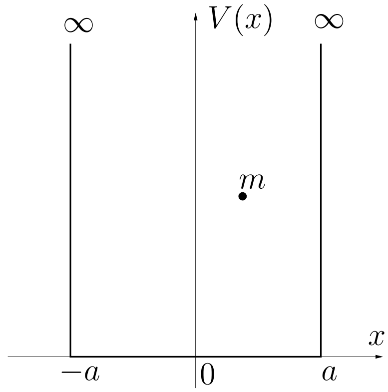

Figure 3: Infinite square well potential.

(a) ***Draw*** the ***wavefunctions*** of the ***three lowest-lying energy eigenstates***.

(b) ***Find*** the ***energies*** of the ***three lowest-lying energy eigenstates*** of the particle.

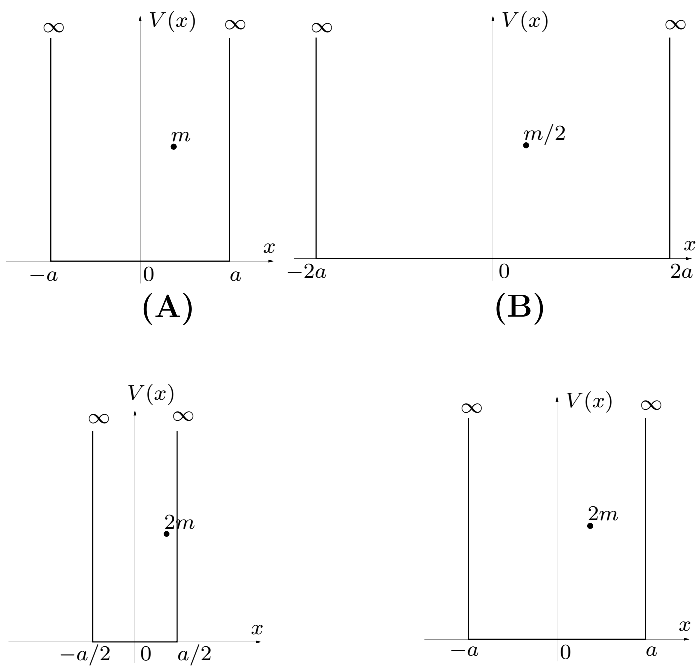

Figure 4: Square wells of different width versus particles of different masses.

(c) ***Which two*** of the configurations shown on Figure 4 have ***the same ground state energies?*** (**"ground state"** = the state of the lowest energy in a given system)

6 Step in a Box

A particle of mass m is trapped in an **infinite square well potential** with a **finite step** in the middle. The width of the well is a and the step (of height V_0) is at $x = a/2$. (See Figure 5.) The height of the step is such that the energy E for each of the lowest two natural states (so-called "eigenstates") is *less than* V_0.

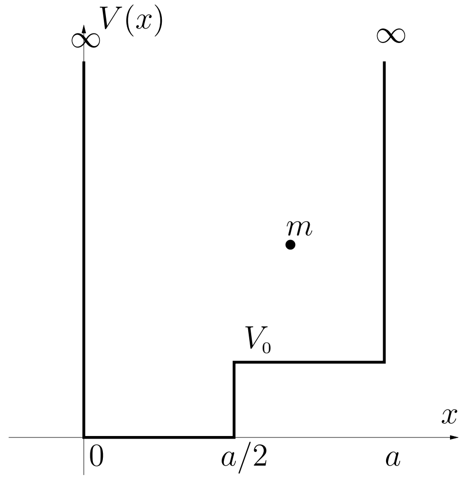

Figure 5: Infinite square well potential.

(a) ***Compute*** the (time-independent) ***wavefunctions*** of the ***two lowest-lying energy eigenstates*** of the particle.

(b) ***Compute*** the ***energies*** of the ***two lowest-lying energy eigenstates*** of the particle.

Hint: Look for solutions of the type $\psi_1(x) = A_1 e^{ikx} + B_1 e^{-ikx}$ in the left region ($x < a/2$) and $\psi_2(x) = A_2 e^{\alpha x} + B_2 e^{-\alpha x}$ in the right region ($x > a/2$). Then, "glue" the wavefunctions smoothly together, i.e., impose the requirements (BCs) that $\psi(x)$ vanish at $x = 0,\ a$, and both $\psi(x)$ and $\dfrac{d\psi}{dx}$ be continuous at $x = a/2$.

7 Finite Well

Now consider a particle of mass m confined in an **finite square well potential** of height V_0 and width a. (See Figure 6.)

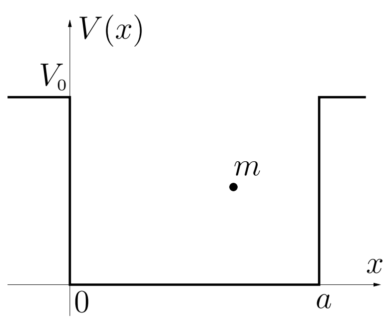

Figure 6: Finite square well potential.

(a) ***Compute*** the (time-independent) ***wavefunctions*** of the ***two lowest-energy states*** of the particle.

Hint: Use the same strategy as in the previous problem. In this case the smoothness requirements (BCs) are that both $\psi(x)$ and $\dfrac{d\psi}{dx}$ be continuous at $x = 0,\ a$.

(b) ***Write down the equations*** that the ***energies*** of the ***two lowest-energy states must satisfy.***

Hint: Some of the boundary conditions above will impose constraints on the wavevectors $k = \sqrt{2mE}/\hbar$ and $k' = \sqrt{2m(E - V_0)}/\hbar$, and thus on the energy E. As a result the energy can only take certain discrete values, solutions to transcendental equations (e.g., $x = \tan x$ is a *transcendental equation*). Such equations can only be solved numerically (or graphically). In this problem you are not required to solve the equations; it is enough just to write them down.

8 Heisenberg uncertainty principle and the ground state energy of the Hydrogen atom

In this problem we will use the *Heisenberg uncertainty principle* and simple considerations involving *energy* and *momentum* to estimate the ground state energy of the Hydrogen atom.

(a) Using the Heisenberg uncertainty principle, ***argue that if the separation of the electron and the nucleus in the Hydrogen atom is of order*** r***, then the momentum of the electron is of order*** $\frac{\hbar}{r}$***.***

(b) ***Write down the total energy of the electron as a function of*** r***.*** Use your result of part (a) to find the *kinetic energy* of the electron, and the Coulomb formula $V(r) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ for its *potential energy.*

(c) ***Find the separation*** r_0 ***at which the energy*** $E(r)$ ***has a minimum. What is the value*** $E_{\min} = E(r_0)$***?*** How do your answers compare with the *Bohr radius,* r_B, and the ground state energy E_1 of Hydrogen, given in *Young & Freedman, eqs. (43-3) and (43-8)*?