

HOMEWORK #1

$$1) \quad x = 3 - 4i \quad y = 3 + 3i$$

$$x = \sqrt{3^2 + 4^2} e^{-i \arctan(4/3)} = 5 e^{-i \arctan(4/3)}$$

$$\arctan(4/3) \approx 0.93$$

$$y = \sqrt{3^2 + 3^2} e^{i \arctan(1)} = 3\sqrt{2} e^{i\pi/4}$$

$$1a) \quad x - y = 3 - 4i - 3 - 3i = -7i$$

$$x - y = -7i = 7 e^{i \arctan(-7/0)} = 7 e^{-i\pi/2}$$

to confirm, $7 e^{-i\pi/2} = 7(\cos(-\pi/2) + i \sin(-\pi/2)) = -7i$ as desired

$$1b) \quad x^2 = (3 - 4i)(3 - 4i) = 9 + 16i^2 - 24i = -7 - 24i$$

$$x^2 = (5 e^{-i \arctan(4/3)})^2 = 25 e^{-2i \arctan(4/3)}$$

$$1c) \quad \frac{y}{x} = \frac{3 + 3i}{3 - 4i} \cdot \frac{3 + 4i}{3 + 4i} = \frac{9 - 12 + 9i + 12i}{9 + 16} = \frac{-3 + 21i}{25} = \frac{-3}{25} + i \frac{21}{25}$$

$$\frac{y}{x} = \frac{3\sqrt{2} e^{i\pi/4}}{5 e^{-i \arctan(4/3)}} = \frac{3\sqrt{2}}{5} e^{i[\pi/4 + \arctan(4/3)]}$$

$$1d) \quad e^x = e^{3-4i} = (e^3) e^{-4i} \quad \text{note this is polar form}$$

$$A = e^3, \phi = -4$$

$$= e^3 \cos(-4) + i e^3 \sin(-4)$$

$$= (e^3 \cos 4) - i (e^3 \sin 4)$$

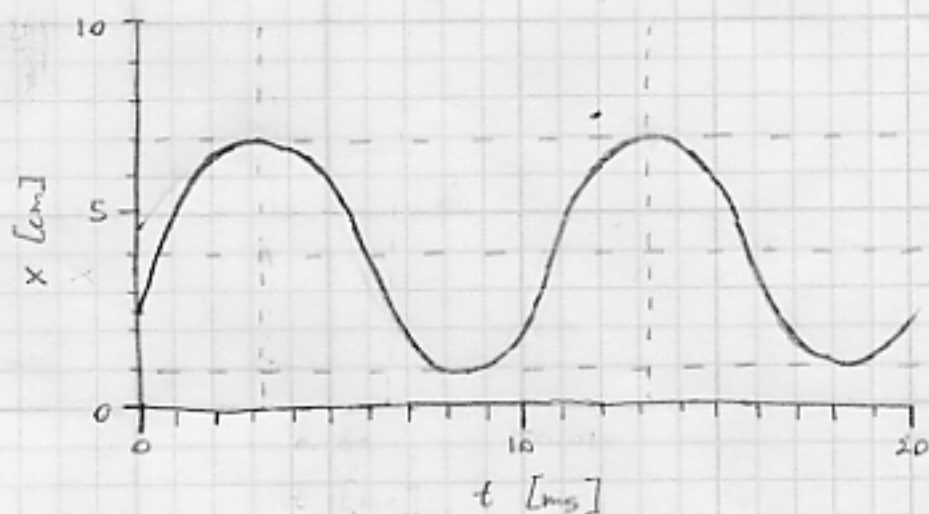
$$\approx -13.13 + 15.20i$$

$$2.1) \quad x(t) = x_0 + A \cos(\omega_0 t + \phi_0)$$

$$= (4.0 \text{ cm}) + (3.0 \text{ cm}) \cos \left[\left(628 \frac{\text{rad}}{\text{sec}} \right) t - \frac{2\pi}{3} \text{ rad} \right]$$

$$\text{so, } A = 3.0 \text{ cm} \quad \omega_0 = 628 \frac{\text{rad}}{\text{sec}} \approx 200\pi \frac{\text{rad}}{\text{sec}} \quad \phi_0 = -\frac{2\pi}{3} \text{ rad}$$

$$f = \frac{\omega_0}{2\pi} = 100 \text{ Hz} \quad T = \frac{1}{f} = 0.01 \text{ sec}$$



$$2.2) \quad x_{\max} = 5 \text{ m}; \quad x_{\min} = -1 \text{ m};$$

$$x_{\text{eq}} = \frac{1}{2}(x_{\max} + x_{\min}) = 2 \text{ m};$$

$$A = \frac{1}{2}(x_{\max} - x_{\min}) = 3 \text{ m};$$

$$T = 1 \text{ s};$$

$$f = \frac{1}{T} = 1 \text{ Hz}; \quad \omega_0 = 2\pi f = 2\pi \frac{\text{rad}}{\text{sec}}$$

$$x(t=0) = 3.5 \text{ m};$$

$$x(t=0) = x_{\text{eq}} + A \cos \phi_0$$

$$3.5 = 2 + 3 \cos \phi_0$$

$$\cos \phi_0 = 0.5$$

$$\phi_0 = \pi/3 \text{ or } 5\pi/3$$

$$v(t=0) = -Aw \sin \phi_0$$

$$v(t=0) > 0 \quad \Rightarrow \quad \sin \phi_0 < 0 \quad \Rightarrow$$

$$\boxed{\phi_0 = \frac{5\pi}{3}}$$