Cornell University

Department of Physics

Phys 214 November 13, 2003

Waves, Optics, and Particles, Fall 2003

Homework Assignment # 10

(Due Thursday, November 20 at 5:00pm *sharp.*)

Agenda and readings for the week of November 17:

Skills to be mastered:

- Be able to compute potential, kinetic and total energy densities and power for a string, given $y(x,t)$
- Understand conservation of energy in wave motion
- Be able to compute electric and magnetic energy densities in an electromagnetic wave
- Be able to compute power flux for a plane E&M wave
- Understand the concept of intensity of an E&M wave and its relation to the power flux and the complex amplitude of the wave.

Lectures and Readings:

Readings marked YF are from the text Young and Freedman, *University Physics*, 10th edition. Readings marked LN are from the course lecture notes to be found at http://people.ccmr.cornell.edu/˜muchomas/P214.

- Lec 23, 11/18 (Tue): Comment on conservation of momentum and energy for other waves. Introduction to Quantum Mechanics, G.P. Thomson experiment.
  Readings: LN "Wave Phenomena III: Transport of momentum and energy," Sec. 4.2; YF 41-1, 41-3.
- Lec 24, 11/20 (Thu): Analysis of G.P. Thomson experiment, measurement of $\hbar$, de Broglie hypothesis.
  Readings: YF 41-2.
- Lec 25, 11/25 (Tue): Heisenberg Uncertainty Principle.
  Readings: YF 41-4
- Lec 26, 12/02 (Tue): Particles in a box; Schrödinger's equation.
  Readings: YF 42-1, 42-2, 42-3, 42-4.
- Lec 27, 12/04 (Thu): Three Nobel Ideas.
  Readings: LN "Feynman Diagrams . . . ," Secs. 3.4, 3.5, 4, 5.

Contents

1 Energy in a Standing Wave

Consider a standing wave on a string with two fixed ends, $x = 0$ and $x = L$. The motion of the string can be described by the equation

$$y(x,t) = A\sin(kx)\cos(\omega t).$$

(a) Find the kinetic energy density, $ke(x,t)$, and the potential energy density, $pe(x,t)$. Is it true that $ke(x,t) = pe(x,t)$? What is the total energy density, $e(x,t)$? Does it depend on time? Is your result consistent with conservation of energy?

(b) Find the power $P(x,t)$.

(c) Show that the equation

$$\frac{\partial e}{\partial t} = -\frac{\partial P}{\partial x}$$

holds for all points on the string at all times. What is the physical meaning of this equation?

(d) The string is oscillating at the *lowest* fundamental frequency. In this part, you will sketch snapshots of some quantities related to the string motion at time t_0 such that $\omega t_0 = \pi/4$. Be sure to clearly mark the axes!

- Sketch the shape of the string, $y(x,t_0)$. On your sketch, indicate with little arrows the direction in which various points on the string are moving.
- Graph the instantaneous power, $P(x,t_0)$.

(e) In which direction is energy propagating in the left half of the string, $0 < x < L/2$? What about the right half, $L/2 < x < L$?

(f) Repeat parts (d) and (e) at time t_1 such that $\omega t_1 = 3\pi/4$.

2 Intensity of Light

Consider a plane, linearly polarized electromagnetic wave. Using complex representation, the fields in the wave are given by

$$\begin{aligned}\vec{E}(x,t) &= \Re(\underline{E}(x)\,e^{-i\omega t})\,\hat{\mathbf{y}},\\ \vec{B}(x,t) &= \frac{1}{c}\Re(\underline{E}(x)\,e^{-i\omega t})\,\hat{\mathbf{z}},\end{aligned}$$

where $\underline{E}(x)$ is the *complex amplitude* of the wave at point x.

(a) Compute the densities of electric field energy and magnetic field energy at x as a function of time t. Simplify your answers so that they do not contain any complex numbers.
HINT: Write the complex amplitude $\underline{E}(x)$ in the polar form and use Euler's formula.

(b) Compute the total energy density at x.

(c) Compute the power flux vector (usually called "Poynting vector" in eletromagnetic theory) $\vec{S}(x,t) \equiv \frac{1}{\mu}\vec{E} \times \vec{B}$. Which direction is energy flowing in? Express the magnitude of the Poynting vector, $S(x,t) \equiv |\vec{S}(x,t)|$, in a form that does not contain any complex numbers.

(d) Show that $S(x,t)$ is a periodic function of time. How is the period of this function related to the period T of the electromagnetic wave itself ($T = 2\pi/\omega$)?

(e) The wavelength of visible light lies in the range between 400 nm (blue) and 700 nm (red). Find the period T of electric and magnetic fields in blue and red light waves.

(f) Intensity $I(x)$ is obtained by averaging $S(x,t)$ over time. Using the results from (d) and (e), explain why $I(x)$ provides a better measure of the "amount of light" seen by a human eye than the quantity $S(x,t)$.

(h) Using your result from part (d), obtain the formula that we have used to study interference,

$$I(x) = \alpha|\underline{E}(x)|^2.$$

What is the value of α in terms of ϵ_0 and μ_0?
HINT: Time-averaged values of $\cos^2(\omega t + \phi_0)$ and $\sin^2(\omega t + \phi_0)$ are both equal to 1/2.

3 Energy in Pulses I

Four identical strings, each with tension $\tau = 9$ N and mass per unit length $\mu = 0.01$ g/m, carry wave pulses traveling to the right. Snapshots of the strings at $t = 0$ are shown in Fig. 1.

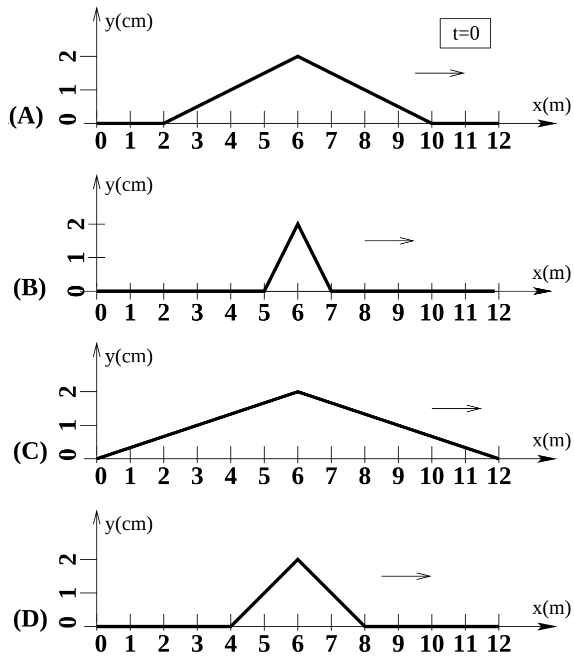

Figure 1: Four pulses for Problem 2.

(a) Compute and graph the kinetic energy density $ke(x, t = 0)$ and the potential energy density $pe(x, t = 0)$ for each of the pulses. Be sure to clearly mark your axes.

(b) Compute and graph the total energy density $e(x, t = 0)$ for each pulse. Which one (A, B, C, or D) has the lowest energy density in the region of the pulse?

(c) Calculate the total energy for each of these pulses. Which one (A, B, C, or D) has the highest total energy?

4 Energy in Pulses II

Two half-infinite strings are connected at $x = 0$, see Fig. 2. The wave speed in the thin string ($x < 0$) is 100 m/sec, and in the thick string ($x > 0$) it's 50 m/sec. A pulse is coming from the left; the shape of the pulse at $t = 0$ is shown in Fig. 2.

(a) Sketch the shape of the strings at $t = 0.1$ sec.
HINT: The impedance of a string is given by $Z = \tau/c$, where c is the wave speed and τ is the tension, common to both strings.

(b) Compute and graph the total energy density at $t = 0$ and $t = 0.2$ sec.

(c) Compute the total energy of the incoming pulse. Repeat the calculation for the reflected and transmitted pulses. Verify conservation of energy.

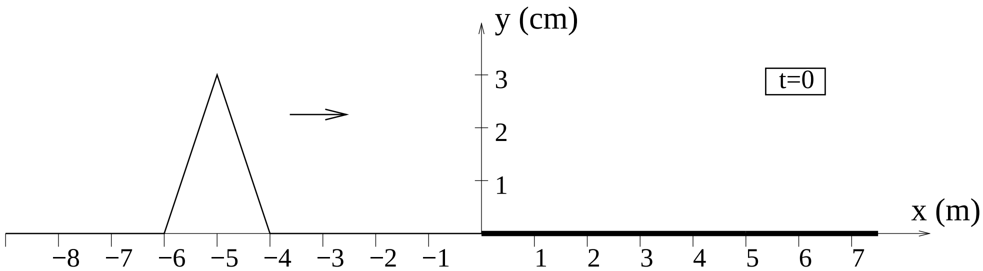

Figure 2: Two half-infinite strings.