Cornell University

Department of Physics

Phys 214 September 3, 2003

Waves, Optics, and Particles, Fall 2003

Homework Assignment # 2

(Due Thursday, September 11 at 5:00pm *sharp*.)

Agenda and readings for the week of September 15:

Skills to be mastered:

- differentiating between the Equation of Motion and a solution of it ;
- verifying a general solution;
- finding particular solutions given initial conditions;
- determining the **complex amplitude** from the initial conditions for a simple harmonic oscillator;
- determining the (real) amplitude and the initial phase from the complex amplitude;
- using the complex representation to solve differential equations.

Lectures and Readings:

Readings marked YF are from the text Young and Freedman, *University Physics*, 10th edition. Readings marked LN are from the course lecture notes to be found at http://people.ccmr.cornell.edu/~muchomas/P214.

- Lec 4, 09/09 (Tue): Damped, driven oscillator; resonance.
  Readings: LN "Simple Harmonic Motion," Sec. 6; YF 13.8
- Lec 5, 09/11 (Thu): Wave equation for the string; standing waves.
  Readings: LN " Intro to Waves: Waves on a String and Standing Waves," Sec. 1-4.3.1; YF 19-1, 19-2, 19-3, 19-4.

Contents

1 Identifying general solutions

Which of the following expressions could be a general solution to the equation of motion for an ideal mass-spring system $-k(x - x_{\rm eq}) = m\frac{d^2x}{dt^2}$? In a quick phrase or two, explain under what condition(s) each of the expressions could be a solution (answer with a phrase like "Yes, a general solution if ω_1=...") or why it can never be:

(a) $x(t) = x_{\rm eq} + A\cos\omega_1 t$;

(b) $x(t) = C + A_1 \sin\omega_1 t + A_2\cos\omega_1 t$;

(c) $x(t) = x_{\rm eq} + A_1 \sin\frac{\omega_1}{2}t + A_2\cos\frac{\omega_1}{2}t$;

(d) $x(t) = x_{\rm eq} + A_1 e^{i\omega_o t} - A_2 e^{-i\omega_o t}$;

(e) $x(t) = x_{\rm eq} + A\cos\left[\omega_o(t - t_o)\right]$;

(f) $x(t) = A_1 \sin\omega_1 t + A_2 \sin\omega_2 t$;

(g) $x(t) = x_{\rm eq} + B\tan\omega_o t + C\sec\omega_o t$;

(h) $x(t) = x_{\rm eq} - A\cos(\phi_o - \omega_o t)$;

(i) $\frac{d^2x}{dt^2} = -\frac{k}{m}(x - x_{\rm eq})$.

2 A slightly different system

A damped oscillator is modeled as a mass m at equilibrium point $x = x_{\rm eq}$ acted on by (1) an ideal spring of spring constant k and (2) a viscous drag force proportional to the velocity: $\vec{F}_{\rm drag} = -bm\vec{v}$.

(a) Derive the equation of motion.
Hint: You can check (especially your signs) against Eq. (13-41) of YF, p. 411. Be careful in comparing, though, because their "b" is actually our "$b \cdot m$" and thus your equation won't look *exactly* like theirs.

(b) Verify that $x(t) = x_{\rm eq} + Ae^{-bt/2}\sin(\omega_1 t + \delta_o)$ is a general solution of the equation of motion. What are the adjustable parameters? What value must ω_1 have in terms of $\omega_o \equiv \sqrt{k/m}$ and b?

Hint: Keep your work organized, and first show

$$\dot{x} = -\frac{b}{2}(x - x_{\rm eq}) + \omega_1 A e^{-bt/2}\cos(\omega_1 t + \delta_o),$$

and then show

$$\ddot{x} = \left(\frac{b^2}{4} - \omega_1^2\right)(x - x_{\rm eq}) - b\omega_1 A e^{-bt/2}\cos(\omega_1 t + \delta_o),$$

before substituting into the equation of motion.

3 A bouncy ride

A truck of mass $m = 1000$ kg *without shock absorbers* (so that there is no damping) rests on suspension springs of a combined effective spring constant $k_{\rm eff}$ such that the full weight of the truck compresses the springs by 2.5 cm. Before $t = 0$ the truck is riding smoothly along (the springs are in their equilibrium position), but at $t = 0$ the truck hits a bump which gives it an initial upward velocity of 10 cm/s from the equilibrium position. You may take the equilibrium position to correspond to $x = 0$.

(a) Find the effective spring constant $k_{\rm eff}$.

(b) Write down the equation of motion of the object.

(c) *Using the complex representation*, find the particular solution $x(t)$ to the object's equation of motion for $t \geq 0$.

(d) What is the amplitude of the resulting oscillations?

4 A different approach to the complex representation

(a) Show, by verifying in the equation of motion and counting the free parameters, that the general solution to the equation of motion of a simple harmonic oscillator (SHO) can be written also as:

$$x(t) = x_{\text{eq}} + \frac{1}{2}\left[\underline{A}e^{i\omega_o t} + \underline{A}^* e^{-i\omega_o t}\right] \ , \tag{1}$$

where $\underline{A}$ is the complex amplitude and $\underline{A}^*$, its complex conjugate.

(b) Express $\underline{A}$ and $\underline{A}^*$ in terms of x_o, v_o, x_{eq}, and ω_o.

5 Complex representation to the rescue

(a) *Use the complex representation* to find a *real general solution* to the equation of motion for the damped harmonic oscillator (see Problem 2). For this problem, you may *assume* that the drag constant b is relatively small: $b < 2\sqrt{k/m}$.

Hint: To make your general solution, use $e^{\underline{\alpha}t}$ where $\underline{\alpha}$ is complex. Be sure to take the real part to get your answer. Compare your answer with the general solution in Problem 2(b) and Eq. (13-42) in YF, p. 412.

(b) Find an expression for and sketch a graph of the particular solution, if $k = 400,000$ N/m, $m = 1000$ kg, $b = 2\ \text{s}^{-1}$, $x_{\text{eq}} = 0$, $x_o = 0$, and $v_o = 10$ cm/s.