Cornell University

Department of Physics

Phys 214 October 8, 2003

Waves, Optics, and Particles, Fall 2003

Homework Assignment # 6

(Due Thursday, October 16 at 5:00pm *sharp*.)

Agenda and readings for the weeks of September 29, October 6:

Skills to be mastered:

- understanding and being able to show that $\vec{E} \perp \vec{B}$ for electric and magnetic fields in the absence of sources (i.e., free charges or currents);
- given the direction of propagation of a an EM wave, being able to obtain the function $\vec{B}(\vec{r}, t)$ from $\vec{E}(\vec{r}, t)$, and vice versa;
- being able to determine the allowed modes for EM standing waves for different types of boundary conditions and to locate the nodal planes of $\vec{E}$ and $\vec{B}$;
- given the polarization of $\vec{E}(\vec{r}, t)$, being able to find the polarization of $\vec{B}(\vec{r}, t)$, and vice versa;
- be able to identify traveling wave solutions $f(x - ct)$ and $g(x + ct)$ and pick out the velocity;
- be able to use the general solution $f(x-ct)+g(x+ct)$ to the wave equation to find particular solutions given the initial conditions (i.e., the string shape and velocity distribution at $t = 0$);
- understanding superposition of *both* displacements and velocities for combinations of left and right pulses.

Lectures and Readings:

Readings marked YF are from the text Young and Freedman, *University Physics*, 10th edition. Readings marked LN are from the course lecture notes to be found at http://people.ccmr.cornell.edu/˜muchomas/P214.

- Lec 12, 10/07 (Tue): Superposition, Chunk velocities, E&M Pol Rule III.
  Readings: LN "Wave Phenomena I," Sec. 4.
- Lec 13, 10/09 (Thu): Reflection at fixed and free boundaries.
  Readings: LN "Wave Phenomena I," Sec. 5; YF 20-1, 20-2, 20-3.
- 10/14 (Tue): **Fall break!!!**
- Lec 14, 10/16 (Thu): Complex representation of reflection/transmission, two-slit interference.
  Readings: "Wave Phenomena II: Interference," Secs. 1-3.2; YF 37-1, 37-2, 37-3, 37-4.

Contents

1 Problem 1

Young & Freedman, Problem 33-5.

2 Problem 2

Young & Freedman, Problem 33-24.

3 Problem 3

Young & Freedman, Problem 33-30.

4 Circular polarization

(a) Show that the expression

$$\vec{E}(x,t) = \hat{y}E_0\cos(\omega t - kx) - \hat{z}E_0\sin(\omega t - kx) \tag{1}$$

is a solution to the wave equation for $\vec{E}(x,t)$.

(b) In our discussion of simple harmonic motion we introduced the *complex representation* by writing the solution to the Equation of Motion as $\Re\mathfrak{e}\left[\underline{A}e^{i\omega t}\right]$, with $\underline{A} = A_r + iA_i$. Using a similar technique, show that the solution (1) can be written in the form

$$\vec{E}(x,t) = \Re\mathfrak{e}\left[(\hat{y} + i\hat{z})\,E_0 e^{i(\omega t - kx)}\right]\ , \tag{2}$$

where $\underline{\hat{\epsilon}} = \hat{y} + i\hat{z}$ is the *complex polarization vector*. Such a wave is called *"circularly polarized"*.

(c) The (equivalent) expressions (1) and (2) represent the $\vec{E}$-field of an EM wave travelling along the x-axis. Derive an equation for the magnetic field of this wave, $\vec{B}(x,t)$ both in real and in complex representation.

(d) Draw two snapshots of the waveform, at times $t = 0$ and $t = T/4$, i.e., for each of these times draw $\vec{E}(x,t)$ and $\vec{B}(x,t)$ (both magnitudes and directions) for at least 5 equally spaced positions in the interval $0 \leq x \leq \lambda$. Can you now tell <u>why</u> this type of E & M wave is called "circularly polarized"?

5 Traveling waves making up another kind of wave

In lecture we learned that the general solution of the wave equation can be written as $y(x,t) = f(x-ct) + g(x+ct)$ (for waves realized on a vibrating string).

Find an explicit expression for the solution $y(x,t)$ for the particular choice $f(u) = g(u) = A\cos(ku)$ of the functions f and g (A is a constant). What kind of wave is this?

Hint: *Rewrite* $A\cos(ku)$ *using the complex representation.*

6 Predicting the future with strings

(*Be sure to ask in section if you have difficulties with this problem!*)

In this question, ignore reflections from the ends of the strings (assume the ends are at $x = \pm\infty$).

(a) At t = 0, a string is plucked. This means the string is initially motionless ($v_y(x,0) = 0$) and has a given initial shape—shown on Figure 1. Consider the form of the general solution of the wave equation $y(x,t) = f(x-ct) + g(x+ct)$. What must the relationship between $f(u)$ and $g(u)$ be, given that the string is initially motionless?

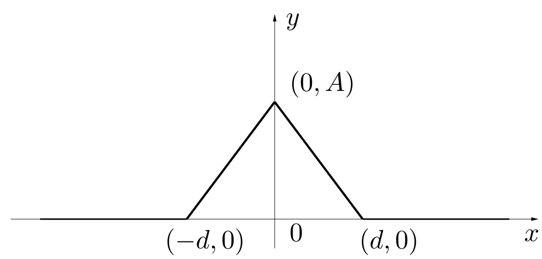

Figure 1: Plucked string.

(b) Sketch what the string looks like at $t = d/c$ and $t = 3d/c$.

(c) At $t = 0$, the hammer of a piano hits a piano string centered at point $x = 0$. In a simplified model, the string is perfectly flat at $t = 0$, but the hammer has given it an initial velocity distribution $v_y(x,0)$. Consider the form of the general solution of the wave equation $y(x,t) = f(x-ct) + g(x+ct)$. What must the relationship between $f(u)$ and $g(u)$ be given that the string is initially flat?

(d) The function $f(u)$ for the piano string is sketched on Figure 2. Use it to sketch the initial velocity distribution $v_y(x,0)$.

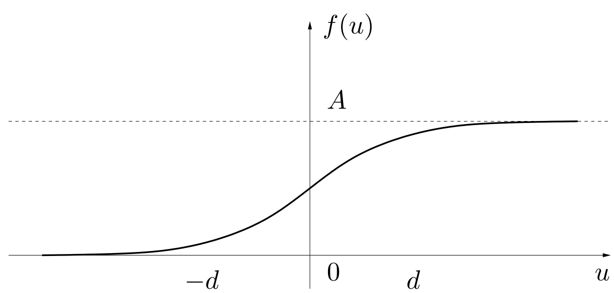

Figure 2: The function $f(u)$ for a piano string.

(e) Sketch what the piano string looks like at $t = d/c$ and at $t = 3d/c$.