1) $\vec{E} = -(3.10\times10^5\ V/m)\,\hat{k}\sin[(2.15\times10^{12}\ rad/s)t - ky] = -E_0\hat{k}\sin(\omega t - ky)$

a) the wave travels in the $+\hat{j}$ direction, since y appears in the sine term and is opposite the t term in sign

b) $\lambda = \frac{c}{f} = \frac{c}{2\pi\omega} \approx 1.40\times10^{-7}$ m

c) $\hat{E}\times\hat{B} = \hat{j} \Rightarrow \hat{B} = \hat{i}$ $\quad B_0 = \frac{1}{c}E_0$

$\vec{B} \approx (1.03\times10^{-3}\,T)\,\hat{i}\sin[(2.15\times10^{12}\ rad/s)t - ky]$

2) 

L = 80 cm
f = 750 MHz
$c = 3\times10^8$ m/s

for a charged particle to remain at rest, it must be at a point where $|\vec{E}| = 0$

we can write $\vec{E}$ using the standing wave expression

$|\vec{E}|(x,t) = E_0\sin(\omega t + \phi_0)\sin(kx + \phi_1)$

with boundary conditions

$|\vec{E}|(0,t) = 0$

$|\vec{E}|(L,t) = 0$

solving theses we find

$\phi_1 = 0$

$k = \frac{n\pi}{L}$

$f = \frac{\omega}{2\pi} = \frac{ck}{2\pi} = \frac{nc}{2L} \Rightarrow n = \frac{2Lf}{c} = 4$

$|\vec{E}|(x,t) = E_0\sin(\omega t + \phi_0)\sin(\frac{4\pi}{L}x) = 0$

$\Rightarrow \sin(\frac{4\pi}{L}x) = 0$

$\Rightarrow \frac{4\pi}{L}x = n\pi \qquad n = 0, 1, 2, \ldots$

$\Rightarrow x = L\frac{n}{4}$

solutions beween 0 and L are

$x = 0, \frac{1}{4}L, \frac{1}{2}L, \frac{3}{4}L, L$

x = 0, 20 cm, 40 cm, 60 cm, 80 cm

3) $\vec{E} = E_0 \hat{j} \sin(\omega t - kx)$ $\quad -\pi \le \phi \le \pi$

$\vec{B} = B_0 \hat{k} \sin(\omega t - kx + \phi)$

$\frac{\partial E_y}{\partial x} = -E_0 k \cos(\omega t - kx) \qquad \frac{\partial B_z}{\partial t} = \omega B_0 \cos(\omega t - kx + \phi)$

we want (33-12)

$\frac{\partial E_y}{\partial x} = \frac{-\partial B_z}{\partial t} \Rightarrow E_0 k \cos(\omega t - kx) = -\omega B_0 \cos(\omega t - kx + \phi)$

in order for this to always be true, we need $\phi = 0$

so,

$-E_0 k \cos(\omega t - kx) = -\omega B_0 \cos(\omega t - kx)$

$E_0 = \frac{\omega}{k} B_0 = cB$

as expected

$\frac{\partial B_z}{\partial x} = -kB_0 \cos(\omega t - kx + \phi) \quad \frac{\partial E_y}{\partial t} = \omega E_0 \cos(\omega t - kx)$

using $\phi = 0$, $E_0 = cB$ in (33-14)

$kB_0 \cos(\omega t - kx) = \omega c \epsilon_0 \mu_0 B_0 \cos(\omega t - kx)$

$\frac{c\omega}{k} \epsilon_0 \mu_0 = 1 \Rightarrow c^2 = \frac{1}{\epsilon_0 \mu_0}$

since we know $c = (\epsilon_0 \mu_0)^{-1/2}$, this is true, so (33-14) is satisfied

4) a) $\hat{E}(x,t) = \hat{y} E_0 \cos(\omega t - kx) - \hat{z} E_0 \sin(\omega t - kx)$

this must satisfy $\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$

$\frac{\partial^2 E}{\partial x^2} = -k^2 E_0 [\hat{y} \cos(\omega t - kx) - \hat{z} \sin(\omega t - kx)]$

$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E_0 [\hat{y} \cos(\omega t - kx) - \hat{z} \sin(\omega t - kx)]$

$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \Rightarrow -k^2 E_0 [\hat{y} \cos(\omega t - kx) - \hat{z} \sin(\omega t - kx)] = \frac{-\omega^2}{c^2} E_0 [\hat{y} \cos(\omega t - kx) - \hat{z} \sin(\omega t - kx)]$

$\Rightarrow -k^2 = \frac{-\omega^2}{c^2}$

$\Rightarrow c^2 = \left(\frac{\omega}{k}\right)^2$

so this satisfies the wave equation if $c = \omega/k$

b) from $e^{i\theta} = \cos\theta + i \sin\theta$

$Re\{e^{i\theta}\} = \cos\theta \qquad Re\{-ie^{i\theta}\} = \sin\theta$

substituting in to our equation,

$\vec{E}(x,t) = \hat{y} E_0 Re\{e^{i(\omega t - kx)}\} - \hat{z} E_0 Re\{-ie^{i(\omega t - kx)}\}$

$= Re\{(\hat{y} + i\hat{z}) E_0 e^{i(\omega t - kx)}\}$

4c) we can apply $\hat{E} \times \hat{B} = \hat{x}$ and $E_0 = cB_0$ to get $\vec{B}(x,t)$

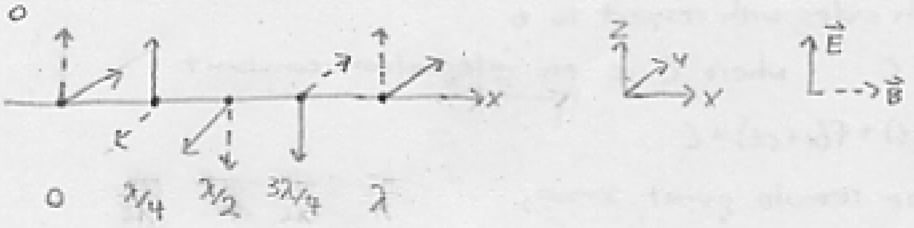
using these in (1),

$$\vec{B}(x,t) = \hat{z}\frac{E_0}{c}\cos(\omega t - kx) + \hat{y}\frac{E_0}{c}\sin(\omega t - kx)$$

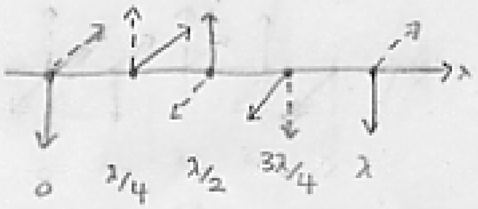
and in (2),

$$\vec{B}(x,t) = \mathrm{Re}\left\{(\hat{z} - i\hat{y})\frac{E_0}{c}e^{i(\omega t - kx)}\right\}$$

d) $t = 0$



$t = T/4$



while $\hat{E}$ and $\hat{B}$ remain orthogonal, their directions rotate in the $+\hat{x}$ direction for increasing x ($-\hat{x}$ for increasing t), tracing out a circle with period λ (in x) or T (in t)

5) $f(u) = g(u) = A\cos(ku)$

$$y(x,t) = f(x - ct) + g(x + ct)$$

$$= A\cos(kx - kct) + A\cos(kx + kct)$$

Using the identity

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$y(x,t) = 2A\cos\left(\frac{2kx}{2}\right)\cos\left(\frac{-2kct}{2}\right)$$

$$= 2A\cos(kx)\cos(\omega t) \qquad \text{since } \omega = ck$$

which is an equation for a standing wave

6) a) at $t=0$, $\frac{\partial y}{\partial t}=0$.

$$\frac{\partial y}{\partial t} = -c\frac{\partial f}{\partial t} + c\frac{\partial g}{\partial t}$$

taking $t=0$ and $x=u$,

$$\frac{\partial y(u)}{\partial t} = -c\frac{\partial f(u)}{\partial t} + c\frac{\partial g(u)}{\partial t} = 0 \quad \Rightarrow \quad \frac{\partial f(u)}{\partial t} = \frac{\partial g(u)}{\partial t}$$

integrating both sides with respect to t

$f(u) = g(u) - C$ where C is an integration constant

$y(x,t) = f(x-ct) + f(x+ct) + C$

but, at some remote point $x \to \infty$,

$y(x,t) \to 0$ and $f(u) \to 0$, so $C = 0$

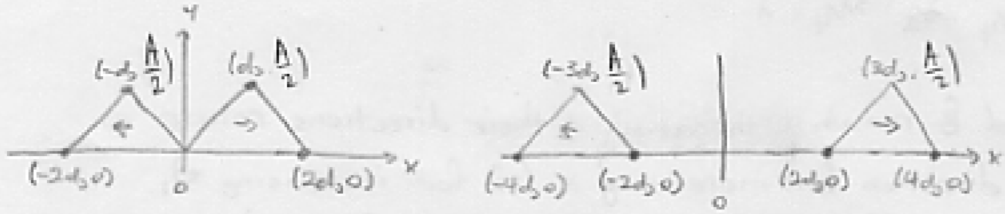
thus we get

$f(u) = g(u)$

and so

$f(u) = \frac{1}{2} y(x,0)$

b)

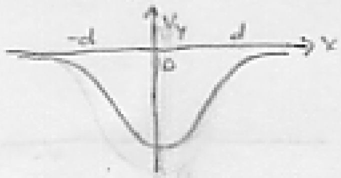


$t = d/c$ $t = 3d/c$

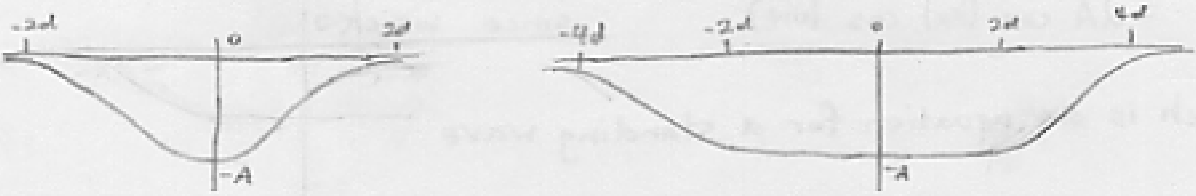
c) $y(x,0) = f(x) + g(x) = 0$

$\Rightarrow f(x) = -g(x)$

d) $v_y(x,0) \propto -\frac{\partial f(u)}{\partial t}$



e)



$t = d/c$ $t = 3d/c$