Cornell University

Department of Physics

Phys 214 October 15, 2003

Waves, Optics, and Particles, Fall 2003

Homework Assignment # 7

(Due Thursday, October 23 at 5:00pm *sharp.*)

Agenda and readings for the weeks of October 20:

Skills to be mastered:

- identifying traveling wave solutions $f(x-ct)$ and $g(x+ct)$ picking out the velocity;
- understanding superposition of *both* displacements and velocities for combinations of left and right pulses.
- using the general solution $f(x-ct)+g(x+ct)$ of the wave equation to find particular solutions;
- finding resulting time history (snapshots at later times, particle histories, velocity distributions) from given initial conditions.
- finding solutions during and after reflection of pulses from fixed, free, and other boundary conditions;
- finding solutions during and after reflection and transmission of pulses from changes in medium.

Lectures and Readings:

Readings marked YF are from the text Young and Freedman, *University Physics*, 10th edition. Readings marked LN are from the course lecture notes to be found at http://people.ccmr.cornell.edu/˜muchomas/P214.

- Lec 15, 10/21 (Tue): Two slit interference pattern
  Readings: LN "Wave Phenomena II: Interference," Sec. 3.2; YF 37-3, 37-4.
- Lec 16, 10/23 (Thu): N slit interference pattern
  Readings: LN "Wave Phenomena II: Interference," Sec. 3.3; YF 38-5 (especially part called "several slits").

Contents

1 To stand or to travel

The velocity of a transverse wave on an infinitely long string is c. At $t = 0$ the string is flat (i.e., $y(x, t = 0) \equiv 0$) and has velocity distribution $v_y(x, t = 0) = V_0 \sin\left(kx + \frac{2\pi}{3}\right)$ where V_0 and k are constants.
Hint: See LN "Wave Phenomena I," Sec. 5.1.

(a) Find the particular solution $y(x, t)$ to the wave equation for the string consistent with the above initial conditions.

(b) Does the solution found in (a) represent a *traveling wave*, a *standing wave*, or *neither*? If a traveling wave, in what direction is it moving? If a standing wave, re-write your solution for part **(a)** in the standing wave form $y(x, t) = A(x)\sin(\omega t)$ and find explicitly the function $A(x)$ in terms of no other quantities than V_0, c, and k.

2 To stand or to travel, part deux

The velocity of a transverse wave on an infinitely long string is c. At $t = 0$ the string has shape given by $y(x, t = 0) = -\dfrac{V_0}{ck}\cos\left(kx + \dfrac{2\pi}{3}\right)$ and velocity distribution $v_y(x, t = 0) = V_0 \sin\left(kx + \dfrac{2\pi}{3}\right)$ where A is a constant.

(a) Find the particular solution $y(x, t)$ to the wave equation for the string consistent with the above initial conditions.

(b) Does the solution found in (a) represent a *traveling wave*, a *standing wave*, or *neither*? Explain. If a traveling wave, in what direction is it moving?

3 Superposition of position and velocities

A pulse (shown on Figure 1) is heading in the x direction at 100 m/s towards the fixed end of a string (at $x = 0$).

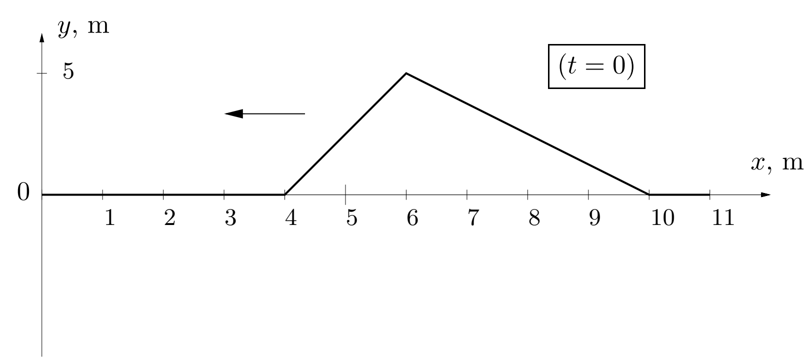

Figure 1: Travelling pulse.

(a) Sketch snapshots at $t = 0.05$ s and $t = 0.06$ s.

(b) Sketch the velocity distributions at $t = 0$, $t = 0.05$ s, and $t = 0.06$ s.

(c) Sketch a particle history for the endpoint ($x = 0$).

(d), (e), (f): Repeat parts (a), (b), and (c) if the end at $x = 0$ is free (in the y direction) instead of fixed.

4 Function transformations

The function $f(u) = (\sin u)e^{-u^2}$ is sketched on Figure 2.

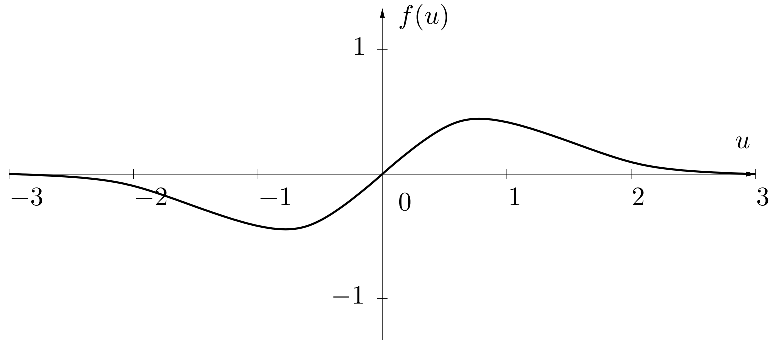

Figure 2: Sketch of the function $f(u) = (\sin u)e^{-u^2}$.

For each of the parts (a) through (d), describe briefly what has happened to the function (compared to $f(u)$) using the following terms:

- translated (shifted)—left, right, up, down, how far;
- reflected—left-right, up-down;
- scaled—contracted, expanded, in what direction (left-right or up-down), by what factor.

Note: Use a plotting program if you wish or sketch by hand. You don't need to hand in the plots or sketches.

(a) $f(u-3)$;

(b) $f(u/2)$;

(c) $f(1-u/2)$;

(d) $5f(2u+1)$

5 Impedance matching

An industrious student decides to experiment with the effects of the damped boundary condition from Problem 6 of Prelim 1,

$$AB\frac{\partial s(x=0,t)}{\partial x} = b\frac{\partial s(x=0,t)}{\partial t},$$

where B is the bulk modulus of the gas in the tube and b the damping coefficient in the shock absorber. (See Figure 3.)

(a) Following the procedure from class and in the lecture notes, determine the form of the reflected pulse $f(u)$ in terms of the incoming pulse $g(u)$ where the general solution is $s(x,t) = f(x-ct) + g(x+ct)$.
Note: Because the shock absorber is on the left of the tube, the *incoming* pulse "g" is traveling from right to left and thus has a "+" sign in front of the factor of c. Your task is to solve for the *reflected* pulse "f", which travels from left to right.
Hint: You should check your answer by investigating the limits $b \to 0$ and $b \to \infty$.

(b) By experimenting with shock absorbers with different damping coefficients b, the student finds—amazingly—that she can send a sound pulse down the tube without any reflection coming back! Find the value(s) of b for which no reflection occurs. Express your answer in terms of the "impedance" $Z \equiv \rho_0 c = B/c$ of the gas in the tube.

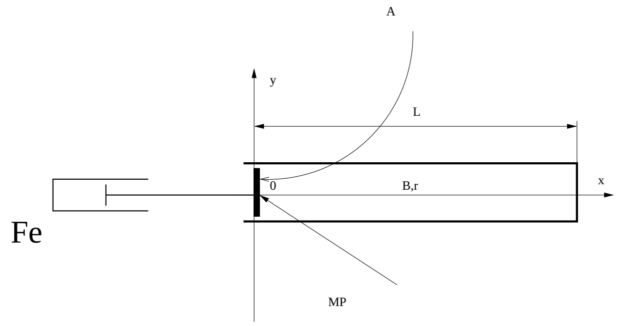

Figure 3: Boundary condition with drag from Prelim 1

<u>Note:</u> Proper termination of waveguides through *impedance matching* like this to avoid unwanted reflections is very important in many electronic communications applications.