1

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HMWRK #7 Sol.

1 (a) for any wave on a string,

$y(x,t) = f(x-ct) + g(x+ct)$

we are told $y(x, t=0) = 0$

so $y(x,t=0) = f(x) + g(x) = 0$ so $\boxed{f(x) = -g(x)}$

and $V_y(x,t=0) = \left.\frac{dy(x,t)}{dt}\right|_{t=0} = -c f'(x-c\cdot 0) + c g'(x+c\cdot 0)$

$= V_0 \sin(kx + \frac{2\pi}{3})$

so $g'(x) - f'(x) = \frac{V_0}{c} \sin(kx + \frac{2\pi}{3})$

so $g(x) - f(x) = -\frac{V_0}{c\cdot k} \cos(kx + \frac{2\pi}{3}) + 0$ ← const. of integration is $\emptyset$ because $g(\infty) = f(\infty) = 0$

we know $f(x) = -g(x)$

so $2\, g(x) = -\frac{V_0}{ck} \cos(kx + \frac{2\pi}{3})$

and $f(x) = \frac{V_0}{2ck} \cos(kx + \frac{2\pi}{3})$

so $\boxed{y(x,t) = \frac{V_0}{2ck}\left(\cos(k(x-ct) + \frac{2\pi}{3}) - \cos(k(x+ct) + \frac{2\pi}{3})\right)}$

(b) note $\cos(A+B) = \cos A \cos B - \sin A \sin B$

so $y(x,t) = \frac{V_0}{2ck}\left(\cos(kx+\frac{2\pi}{3})\cos(-ckt) - \sin(kx+\frac{2\pi}{3})\sin(-ckt) - \cos(kx+\frac{2\pi}{3})\cos(ckt) + \sin(kx+\frac{2\pi}{3})\sin(ckt)\right)$

$= \frac{V_0}{2ck}\left(2\sin(kx + \frac{2\pi}{3})\sin(ckt)\right)$

note $ck = w$

$y(x,t) = \boxed{\frac{V_0}{ck} \sin(kx + \frac{2\pi}{3}) \sin(wt)}$

This is a standing wave form.

note $t=0 \rightarrow y = 0$ and $V_y(x, t=0) = V_0 \sin(kx + \frac{2\pi}{3})$

Checked ✓

2 (a) $y(x, t=0) = f(x) + g(x) = -\frac{V_0}{ck}\cos\left(kx + \frac{2\pi}{3}\right)$ (1)

and $-cf'(x) + cg'(x) = V_0 \sin\left(kx + \frac{2\pi}{3}\right)$

Integrating: $-f(x) + g(x) = -\frac{V_0}{ck}\cos\left(kx + \frac{2\pi}{3}\right) + 0$

(2) $\qquad f(\infty) = g(\infty) = 0$

adding (1) and (2):

$(1) + (2) = 2g(x) = -\frac{2V_0}{ck}\cos\left(kx + \frac{2\pi}{3}\right)$

$$\boxed{\begin{array}{l} g(x) = -\frac{V_0}{ck}\cos\left(kx + \frac{2\pi}{3}\right) \\ f(x) = 0 \end{array}}$$

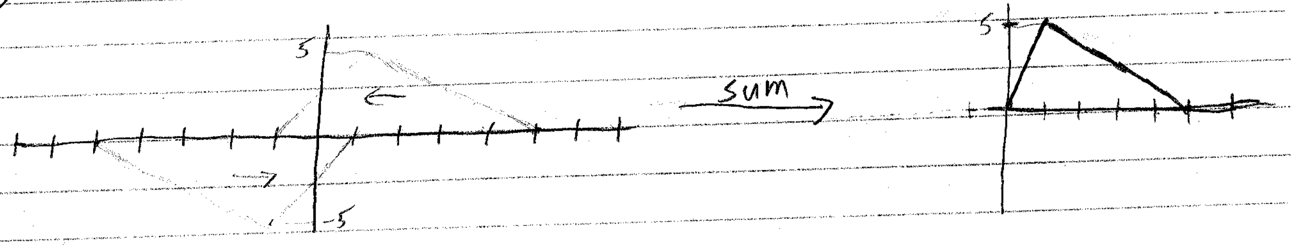
so $\boxed{y(x,t) = -\frac{V_0}{ck}\cos\left(k(x+ct) + \frac{2\pi}{3}\right)}$

Check: $y(x, t=0) = -\frac{V_0}{ck}\cos\left(kx + \frac{2\pi}{3}\right)$ ✓

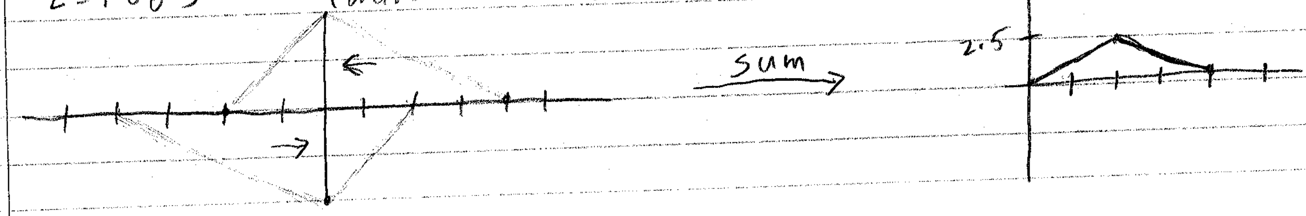
$V(x, t=0) = V_0 \sin\left(kx + \frac{2\pi}{3}\right)$ ✓

(b) This is a traveling wave moving in the $-x$ direction.

3 (a) $t = .05s$ (wave has moved 5m)

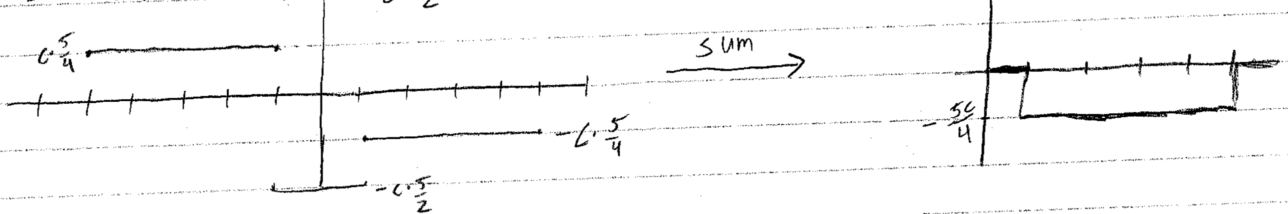


$t = .06\,s$ (wave has moved 6m)

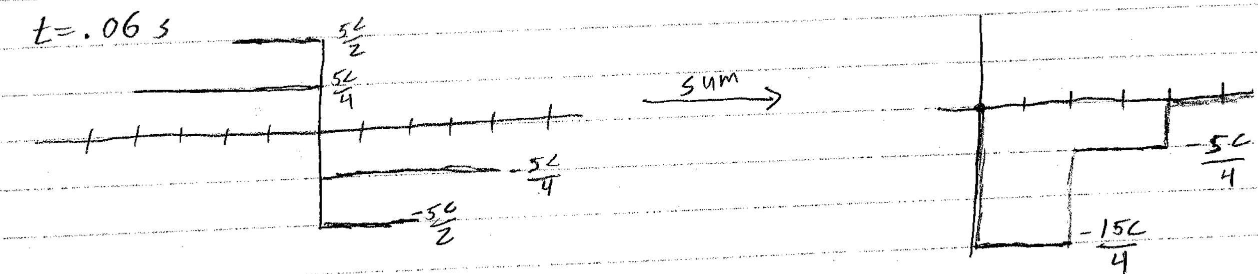


(b) for left moving pulse, $v = \frac{dy}{dt} = c\,\frac{dy}{dx} = c \cdot slope$ } by Pulse eq.
for right moving pulse, $v = -c \cdot slope$

$t = .05\,s$

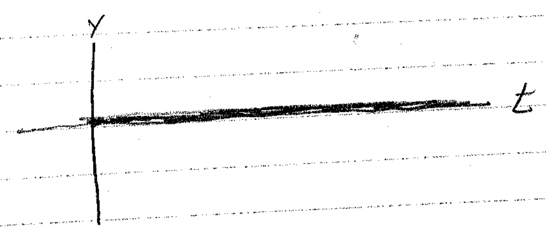


$t = .06\,s$

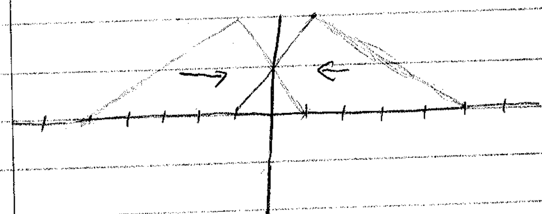


(c) it's a fixed end!

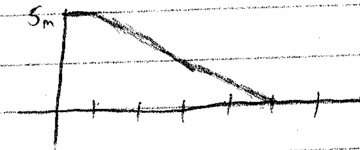
$y(x=0, t) = 0$



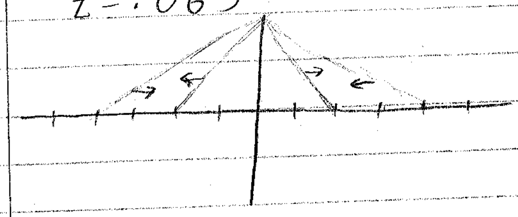
3 (d) t = .05 s



sum →



t = .06 s



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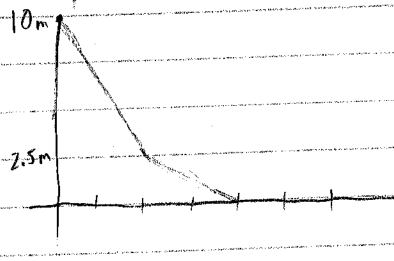
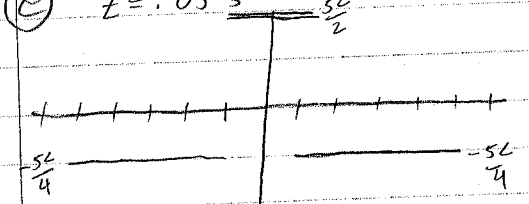
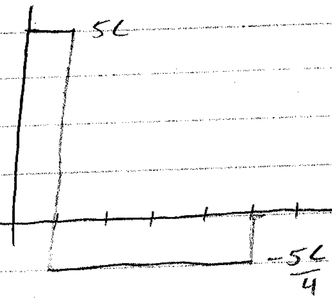


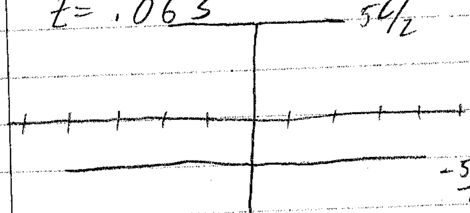
(e) t = .05 s



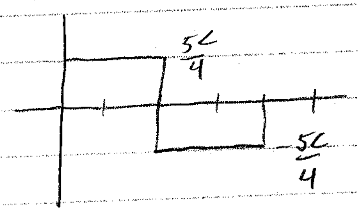
sum →



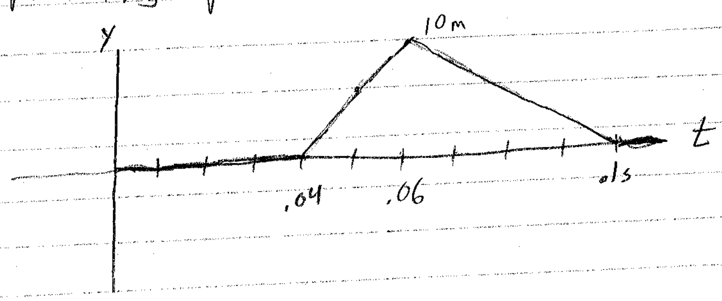
t = .06 s



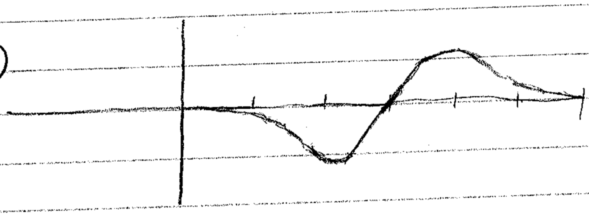
sum →



(f) plotting points:

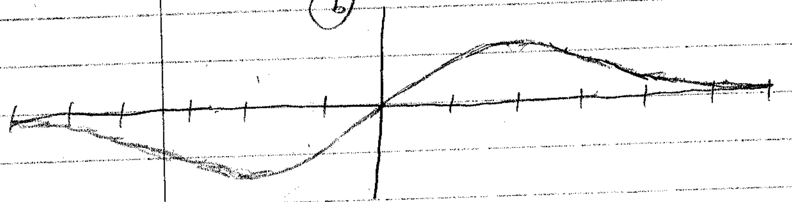


4 (a)



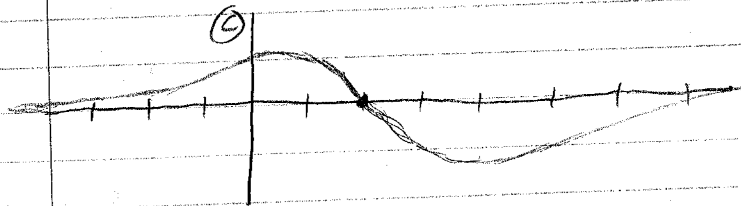
Shifted to right 3

(b)

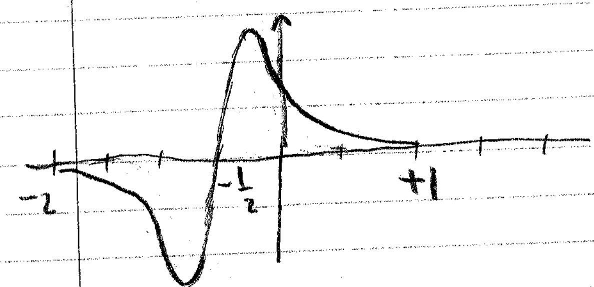


streched to twice what it was.

(c)



streched 2X, shifted right 2, inverted left - right



Amplitude is 5X what it was. squished 2X, shifted 1/2 left.

5 (a) $s(x,t) = f(x-ct) + g(x+ct)$

so $AB\left(f'(-ct) + g'(ct)\right) = b\left(-cf'(-ct) + cg'(ct)\right)$

note $x=0 \longrightarrow$

so $AB\left(f'(u) + g'(-u)\right) = b\left(-cf'(u) + cg'(-u)\right)$

where $u = -ct$

integrating:

$AB\left(f(u) - g(-u)\right) = b\left(-cf(u) - cg(-u)\right) + \text{const.}$

$f(\infty) = g(\infty) = 0$ $u = \infty \Rightarrow$ $AB(0+0) = b(0+0) + \text{const.}$ so const $= 0$

These negative signs are needed to make the derivative work. consider the chain rule...

solving for $f(u)$:

$$\boxed{f(u) = g(u)\,\frac{(AB - bc)}{(AB + bc)}}$$

note $b=0$ is like a free end to the pipe.
in this case, $f(u) = g(-u)$ ✓

and $b=\infty$ is like a closed end, and
$f(u) = -g(u)$ ✓

(b) note if $b = \frac{AB}{C}$, $f(u)=0$, no reflection.
so $\boxed{b = AZ}$ is the special value.