Cornell University

Department of Physics

Phys 214 November 9, 2003

Waves, Optics, and Particles, Fall 2003

Homework Assignment # 9

(Due Thursday, November 13 at 5:00pm *sharp.*)

Agenda and readings for the week of November 10:

Skills to be mastered:

- Understand the pattern produced by N slit interference
- Understand the locations of the principal maxima, lesser maxima, and minima in an N slit pattern
- Understand the concept of *width* of a maximum in an interference pattern
- Understand the properties of the interference pattern produced by diffraction on a finite slit.

Readings marked YF are from the text Young and Freedman, *University Physics*, 10th edition. Readings marked LN are from the course lecture notes to be found at http://people.ccmr.cornell.edu/~muchomas/P214.

- Lec 21, 11/11 (Tue): Differential equation form for conservation laws; conservation of momentum. **Readings: LN "Wave Phenomena III: Transport of momentum and energy," Secs. 1, 2, 3.**
- Lec 22, 11/13 (Thu): Conservation of energy, power, special cases for traveling waves. **Readings: LN "Wave Phenomena III: Transport of momentum and energy," Sec. 4.**

Contents

1 Using Interference to Study Small Objects

A system of several identical parallel narrow slits is illuminated with blue light of wavelength $\lambda = 450$ nm. The interference pattern shown in Fig. 1 is observed on a distant screen.

(a) How many slits are there?

(b) What is the separation between the slits? Compare your answer to the typical thickness of a human hair, about 0.01 cm.

(c) If the screen is placed 1 m away from the slit system, what is the distance on the screen between the neighbouring principal maxima of light intensity? Between neighbouring minima?

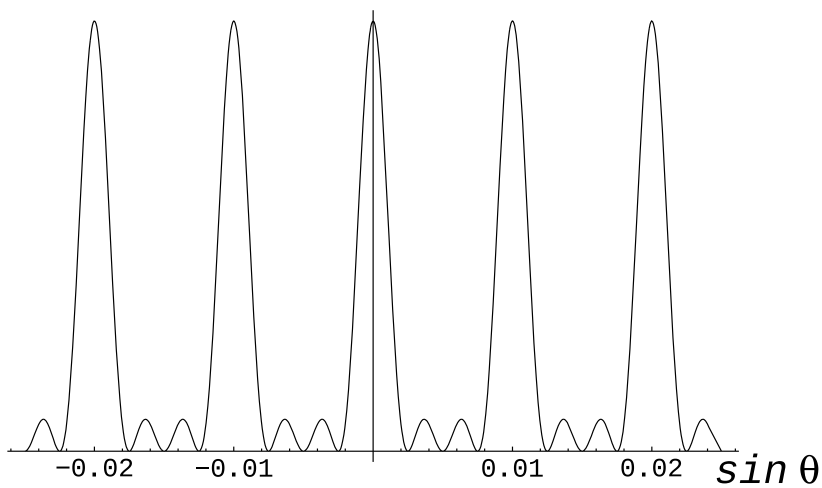

Figure 1: Interference pattern observed on a distant screen.

2 Reflection Grating

Reflection grating consists of an array of N equally spaced, identical narrow reflecting grooves or ridges on a non-reflecting surface. The spacing between the grooves is $d = 10^{-6}$ m. Light from a distant source falls *normally* (that is, perpendicularly to the surface) on the grating. A distant observer observes light reflected at angle θ, see Fig. 2.

(a) If the source emits blue light (wavelength $\lambda = 450$ nm), what values of the angle θ correspond to principal maxima? Does the answer depend on N?

(b) If the source emits red light (wavelength $\lambda = 700$ nm), what values of θ correspond to principal maxima?

(c) Using the above results, explain why the surface of a compact disc looks "rainbow-colored" when seen from an angle.

HINT: Natural light is a mixture of waves with wavelength spanning the whole spectrum from blue to red.

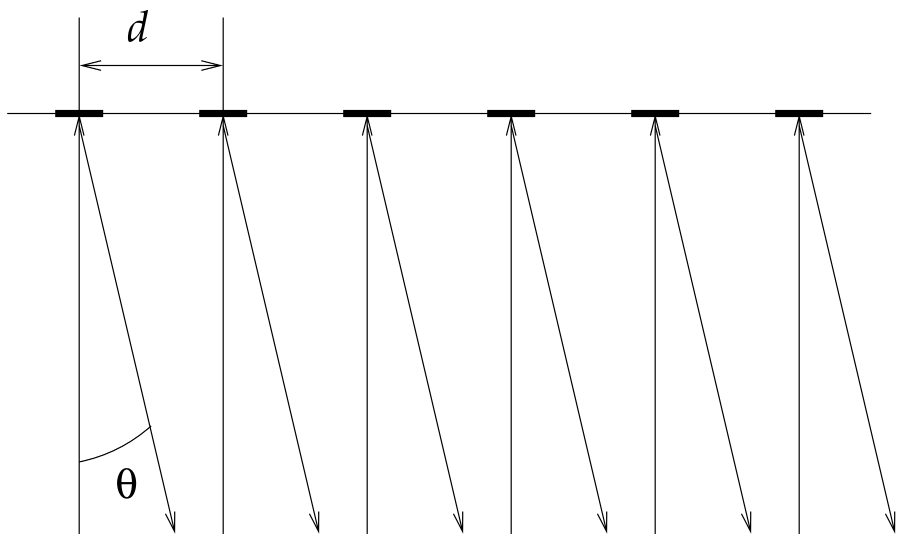

Figure 2: Reflection grating.

3 Resolving Power of the Grating

Consider once again the reflection grating in Fig. 2. It is now illuminated with a mixture of light with two wavelengths, λ_1 and λ_2.

(a) Find the positions of the principle maxima (in terms of $\sin\theta$) for each λ. Sketch (on the same plot) the interference patterns for each λ. (You can choose any value of $N \geq 3$ for your sketches.) Using the sketches, explain why the grating can be used to separate, or "resolve", the different wavelengths.

(b) The *width* of a principal maximum can be defined as the distance (in terms of $\sin\theta$) between the two minima neighbouring the maximum. Find the width of the principle maxima for each λ.

(c) Consider the situation when the two wavelengths are very close to each other, $\lambda_2 = \lambda_1 + \delta$. What is the minimal value of δ for which these waves can still be resolved?

(d) Sketch (on the same plot) the interference patterns for two wavelengths that are too close to each other to be resolved. (Again, choose any $N \geq 3$.)

4 Interference with an Off-Axis Source

Consider again the reflecting grating of problems 2, 3. The distant source has been moved so that light no longer falls on the grating normally, but at an angle α, as shown in Fig. 3. In this problem, you will learn how this change affects the interference pattern seen by a distant observer.

(a) Consider two waves. "Wave 0" has been emitted by the source, reflected by the leftmost reflecting groove ("groove 0"), and observed by a distant observer viewing the grating at angle θ. "Wave 1" has been emitted by the same source, reflected by the second groove on the left ("groove 1"), and then observed by the same observer. What is the difference in the paths travelled by waves 0 and 1? What is their *phase* difference when they reach the observer?

NOTE: In this problem, please keep λ and d as variables – do not use the values given in problem 2!

(b) Repeating part (a), find the phase difference between wave 0 and all the other waves seen by the observer (wave 2, wave 3, ..., wave $N-1$.)

(c) Find the intensity measured by the observer as a function of θ. (Your answer can also contain α, λ, d, N, and I_0 - the intensity the observer would have measured if there were only a single reflecting groove.)

(d) Sketch the interference pattern, $I(\sin\theta)$. Explain in a brief sentence or two how this pattern differs from the previously studied case, $\alpha = 0$.

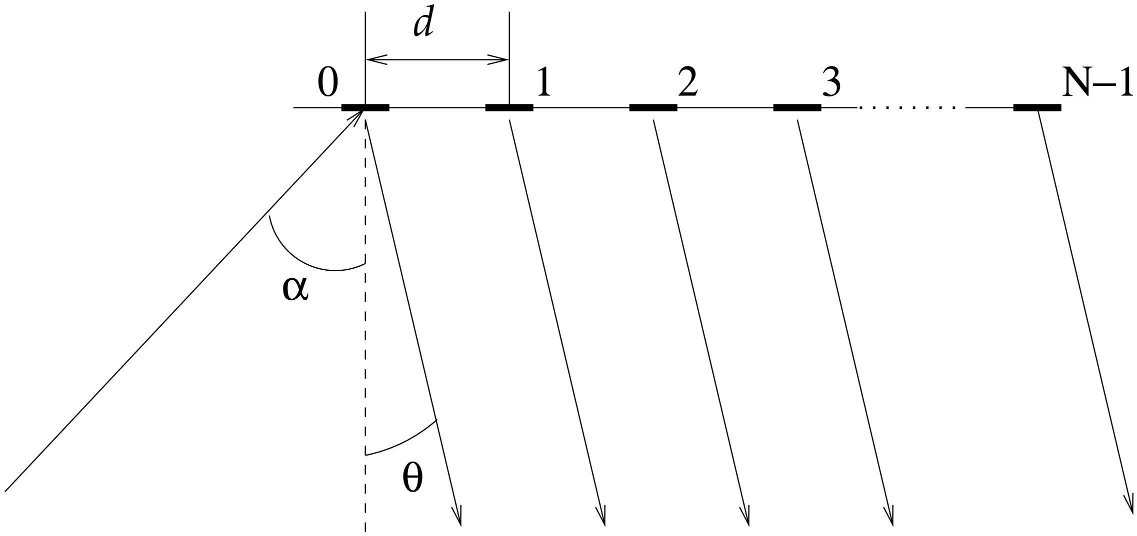

Figure 3: Off-axis interference with a reflection grating.

5 Alone in the Dark

You're in a dark, soundproof house at night. The roof is 7 m high, and has a long straight crack in it which is only 0.05 mm wide. Apart from the crack, the walls of the house do not let in any light or sound.

(a) A bright star at 90 degrees above the horizon, see Fig. 4, emits light of wavelength $\lambda = 500$ nm. Where in the room should you stand to be able to see the star? Give your answer in terms of the horizontal distance x between your eye and the crack. Assume you're 2 m tall.

HINT: You can assume that you do not see anything once you're outside the central intensity maximum of the diffraction pattern. You should be able to invent a sensible definition of the width of that maximum.

(b) If an airplane passes above the house, where in the room should you stand to be able to hear it? You can assume that the sound produced by the airplane has frequency $f = 100$ Hz, and the speed of sound in air is 330 m/sec.

(c) What is the crucial difference between light and sound that makes the answers to (a) and (b) so different?

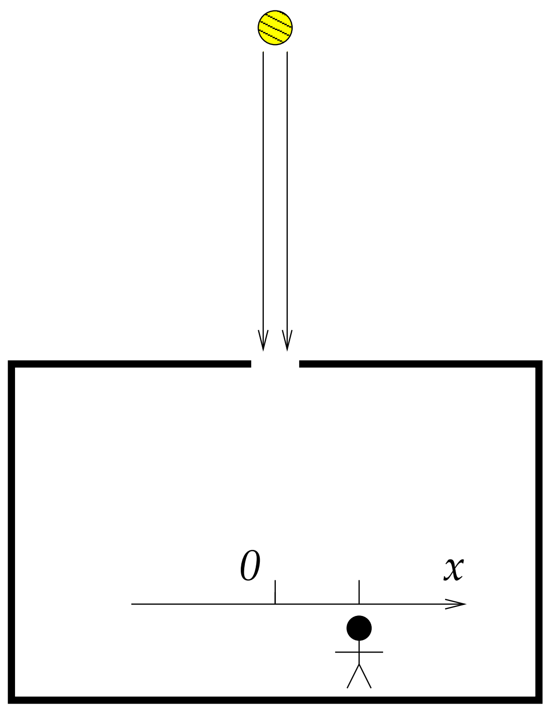

Figure 4: The dark house with a crack in the roof.