Physics 214
Homework # 9

1a) $I = I_0 \dfrac{\sin^2\left(\frac{N\Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)}$ has minima when $\frac{N\Delta\phi}{2} = m\pi$ for

m an integer, <u>except</u> when $\frac{m}{N}$ is also an integer, $\Rightarrow$
$m = 1, 2, \ldots, N-1$ label the minima between the adjacent principal
maxima at $m = 0$ and $m = N$. Since Fig. 1 shows 3 minima
between adjacent principal maxima, there must be 4 slits.

1b) Fig. 1 shows adjacent principal maxima at $\sin\theta = 0$
and $\sin\theta = 0.01$. These principal maxima occur at
$\frac{kd\sin\theta}{2} = 0$ and $\frac{kd\sin\theta}{2} = \pi \Rightarrow \frac{2\pi d(0.01)}{2\lambda} = \pi \Rightarrow d = \frac{\lambda}{0.01}$
$= 4.5 \times 10^{-5}$ m when $\lambda = 450$ nm. So, d is about half the
thickness of a human hair.

1c) The angular distance between adjacent principal maxima
is $\theta \approx 0.01$ rad. Since this is a small angle, the distance
on the screen between principal maxima is just $r\theta =$
$(1\text{m})(0.01\text{ rad}) = 1$ cm, where r is the distance to the
screen, and θ is the angle subtended in radians.

The distance on the screen between neighboring minima
is $\frac{1}{4}$ cm, since $\theta = 0.025$ (from Fig. 1 or part (a)).

2a) $\lambda = 450$ nm $d = 10^{-6}$ m

We have principal
maxima when the reflections
interfere constructively $\Rightarrow kd\sin\theta = 2\pi(\text{integer}) \Rightarrow$
$\sin\theta = (\text{integer})\frac{\lambda}{d} = (\text{integer})(0.45)$, so $\theta = \sin^{-1}((\text{integer})(0.45))$

The answer does not depend on N. Only $n = \pm 1, \pm 2$ possible
since $\sin\theta \le 1$ and
$\theta_1 = \pm 26.7°$, $\theta_2 = \pm 64.2°$, $\theta = 0$.

2b) Plugging in $\lambda = 700\,nm$ into the formula found above, we have $\theta = \sin^{-1}((\text{integer})(0.70))$. only $\theta = \pm 44.4°$ and $\theta = 0$

2c) Comparing (a) and (b), we see that a reflection grating spreads out light according to wavelength. A compact disc has pits with a spacing on the order of 10^{-6} m, and acts as a reflection grating, separating natural white light into its components.

3a) The principal maxima are at $\sin\theta_1 = (\text{integer})\frac{\lambda_1}{d}$ and $\sin\theta_2 = (\text{integer})\frac{\lambda_2}{d}$. With $N=3$, we have

$$I_1(\theta) = I_0\left(\frac{\sin\left(\frac{3\pi\sin\theta}{\lambda_1/d}\right)}{\sin\left(\frac{\pi\sin\theta}{\lambda_1/d}\right)}\right)^2,$$ and similarly for λ_2.

The plot attached is for λ_1 and $\lambda_2 = 1.5\lambda_1$; different wavelengths have principal maxima at different angles, they are "resolved".

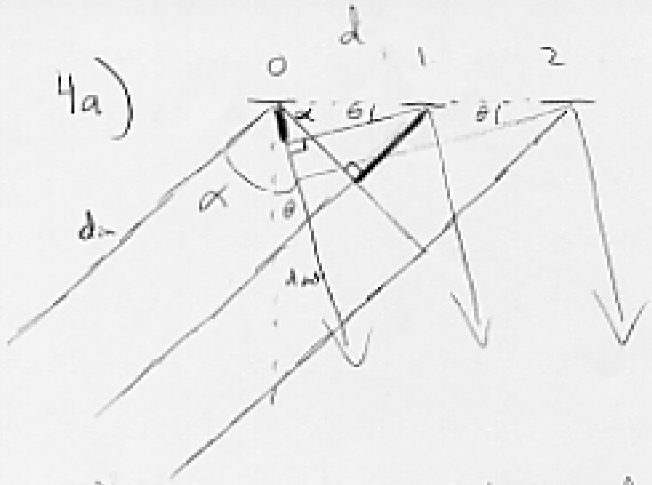
3b) In terms of $\sin\theta$, principal maxima occur at integer multiples of λ/d. Minima occur at integer multiples of λ/Nd, except when this is an integer multiple of λ/d. So, in terms of $\sin\theta$, the distance between 2 minima neighboring a principal maximum is $\frac{2\lambda}{Nd}$. The widths of principal maxima are $\frac{2\lambda_1}{Nd}$ and $\frac{2\lambda_2}{Nd}$, respectively.

3c) We will consider wavelengths to be resolved if the distance between the centers of the principal maxima is at least as great as $\frac{1}{2}$ of the width of the narrower maximum: $\frac{\lambda_1+\delta}{d} - \frac{\lambda_1}{d} \geq \frac{\lambda_1}{Nd} \Rightarrow \delta \geq \frac{\lambda_1}{N}$

for first principal maximum

3d) Please see the attached plot for $N=3$, $\lambda_2 = 1.25\lambda_1$

3c) For nth principal maximum $n\frac{\lambda_1+\delta}{d} - n\frac{\lambda_1}{d} \geq \frac{\lambda_1}{Nd} \Rightarrow \delta \geq \frac{\lambda_1}{nN}$

4a) 

Path 0 − Path 1 = $d(\sin\theta - \sin\alpha)$

The phase difference is $\frac{2\pi}{\lambda} d(\sin\theta - \sin\alpha)$

4b) Using the picture above and similar reasoning, the phase difference between wave N−1 and wave 0 is $-(N-1)\frac{2\pi d}{\lambda}(\sin\theta - \sin\alpha)$.
(There is a sign difference from the question above.)

4c) The phase difference $\Delta\phi$ between neighboring reflections is $-\frac{2\pi}{\lambda}d(\sin\theta - \sin\alpha)$, so we have $I = I_0 \frac{\sin^2\left(\frac{N\Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)}$

Derivation see lecture notes on Interference and Diffraction section 3.3.1

$$= I_0 \frac{\sin^2\left(-\frac{N\pi}{\lambda}d(\sin\theta - \sin\alpha)\right)}{\sin^2\left(-\frac{\pi}{\lambda}d(\sin\theta - \sin\alpha)\right)} = I_0 \frac{\sin^2\left(\frac{N\pi d}{\lambda}(\sin\theta - \sin\alpha)\right)}{\sin^2\left(\frac{\pi d}{\lambda}(\sin\theta - \sin\alpha)\right)}$$

4d) Please see the attached plot, drawn for $\alpha = 30°$.

5a) $I(\theta) = I_{max} \frac{\sin^2\left(\frac{ka\sin\theta}{2}\right)}{\left(\frac{ka\sin\theta}{2}\right)^2}$ The minima next to the central max are at $\frac{ka\sin\theta}{2} = \pm\pi$

$$\Rightarrow \sin\theta = \pm\frac{\lambda}{a} = \pm\frac{5\times10^{-7}\,m}{5\times10^{-5}\,m} = \pm 0.01 \Rightarrow \theta = \pm 0.01 \text{ rad}.$$

Let the distance between your eye and the crack be 5 m. (5 m)(0.01 rad) = 5 cm, so you should stand within ±5 cm of x = 0. Note that <u>at</u> ±5 cm, you are at a minimum, so you should stand within a smaller region, say ± 2.5 cm.

5b) With f = 100 Hz, c = 330 m/s, λ = 3.3 m. $\lambda/a \gg 1 \Rightarrow$ the central max spreads over 180°, so you can hear the airplane in the whole room — except it won't be very loud.

5c) The relevant difference between light and sound in this problem is the wavelength.

4c, continued) We can derive the intensity formula used.

$$I = \left| \sqrt{I_0}\, e^{ik(d_{in} + d_{out})} + \sqrt{I_0}\, e^{ik(d_{in} + d \sin\alpha + d_{out} - d\sin\theta)} + \dots + \sqrt{I_0}\, e^{ik(d_{in} + (N-1) d \sin\alpha + d_{out} - (N-1) d \sin\theta)} \right|^2$$

$$= I_0 \left| 1 + e^{ikd(\sin\alpha - \sin\theta)} + \dots + e^{ikd(N-1)(\sin\alpha - \sin\theta)} \right|^2$$

$$1 + r + \dots + r^{N-1} = \frac{1 - r^N}{1 - r}$$

$$I = I_0 \left| \frac{1 - e^{ikd(\sin\alpha - \sin\theta)N}}{1 - e^{ikd(\sin\alpha - \sin\theta)}} \right|^2$$

$$= I_0 \left(\frac{1 - e^{ikd(\sin\alpha - \sin\theta)N}}{1 - e^{ikd(\sin\alpha - \sin\theta)}} \right) \left(\frac{1 - e^{-ikd(\sin\alpha - \sin\theta)N}}{1 - e^{-ikd(\sin\alpha - \sin\theta)}} \right)$$

$$= I_0 \left(\frac{2 - e^{ikd(\sin\alpha - \sin\theta)N} - e^{-ikd(\sin\alpha - \sin\theta)N}}{2 - e^{ikd(\sin\alpha - \sin\theta)} - e^{-ikd(\sin\alpha - \sin\theta)}} \right)$$

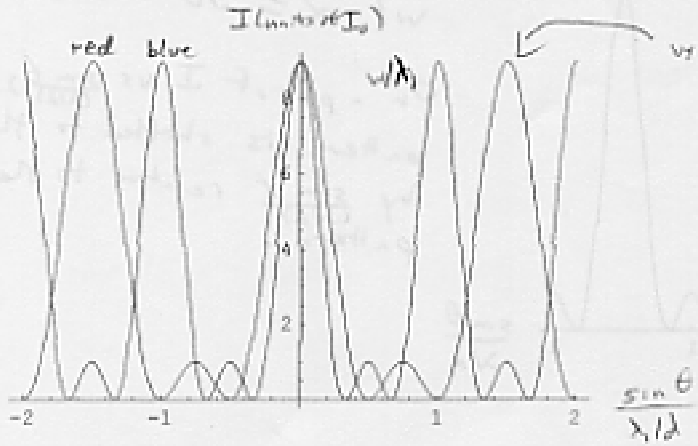
$$= I_0 \left(\frac{2 - 2\cos(kd(\sin\alpha - \sin\theta)N)}{2 - 2\cos(kd(\sin\alpha - \sin\theta))} \right)$$

$$= I_0 \left(\frac{1 - \cos(kd(\sin\alpha - \sin\theta)N)}{1 - \cos(kd(\sin\alpha - \sin\theta))} \right)$$

$$= I_0 \frac{\sin^2\left(\frac{Nkd(\sin\alpha - \sin\theta)}{2}\right)}{\sin^2\left(\frac{kd(\sin\alpha - \sin\theta)}{2}\right)} = I_0 \frac{\sin^2\left(\frac{Nkd(\sin\theta - \sin\alpha)}{2}\right)}{\sin^2\left(\frac{kd(\sin\theta - \sin\alpha)}{2}\right)}$$

3a)

```
In[2]:= Plot[{((Sin[3*Pi*x]) / (Sin[Pi*x]))^2,
    ((Sin[3*Pi*x*(1/1.5)]) / (Sin[Pi*x*(1/1.5)]))^2}, {x, -2, 2}]
```



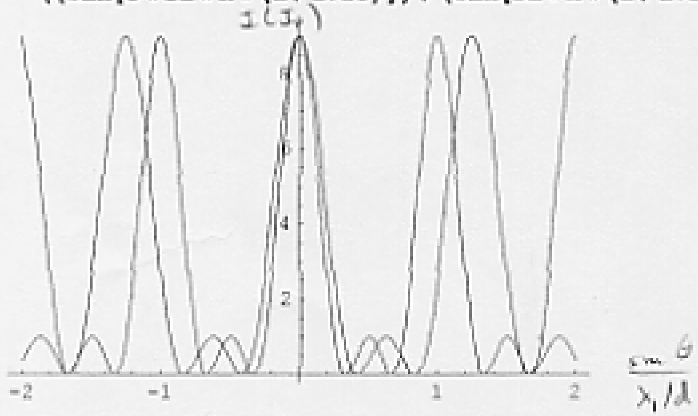
w/ $\lambda_2 = 1.5\lambda_1$

Note that $\frac{\sin\theta}{\lambda/d} = n$

```
Out[2]= - Graphics -
```

3d)

```
In[3]:= Plot[{((Sin[3*Pi*x]) / (Sin[Pi*x]))^2,
    ((Sin[3*Pi*x*(1/1.25)]) / (Sin[Pi*x*(1/1.25)]))^2}, {x, -2, 2}]
```



For $N=3$, $\lambda_2 = 1.25\lambda_1$

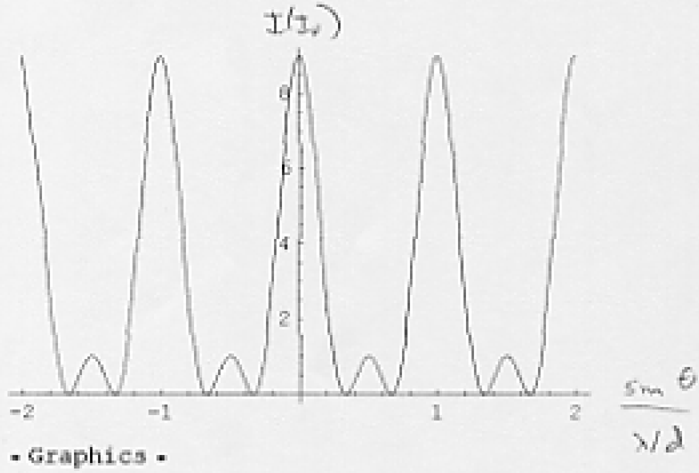
$\Rightarrow \delta = .25\lambda_1 < \frac{\lambda_1}{3}$

$\Rightarrow$ can't resolve.

```
Out[3]= - Graphics -
```

4d)

```
In[5]:= Plot[((Sin[3*Pi*x]) / (Sin[Pi*x]))^2, {x, -2, 2}]
```



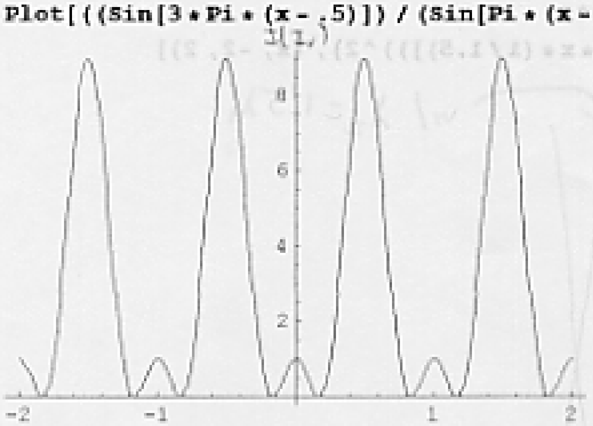
$\alpha = 0$

```
Out[5]= - Graphics -
```


$\rightarrow$

In[6]:= `Plot[((Sin[3 * Pi * (x - .5)]) / (Sin[Pi * (x - .5)]))^2, {x, -2, 2}]`

w/ $\alpha = 30^\circ$



$\frac{\sin\theta}{\lambda/d}$

On a plot of I vs $\frac{\sin\theta}{(\lambda/d)}$, the pattern is shifted to the right by $\frac{\sin\alpha}{(\lambda/d)}$ relative to the $\alpha=0$ pattern.

Out[6]= - Graphics -