CORNELL UNIVERSITY

Department of Physics

Physics 214 Prelim II Fall 2004

NAME:

SECTION:

Instructions

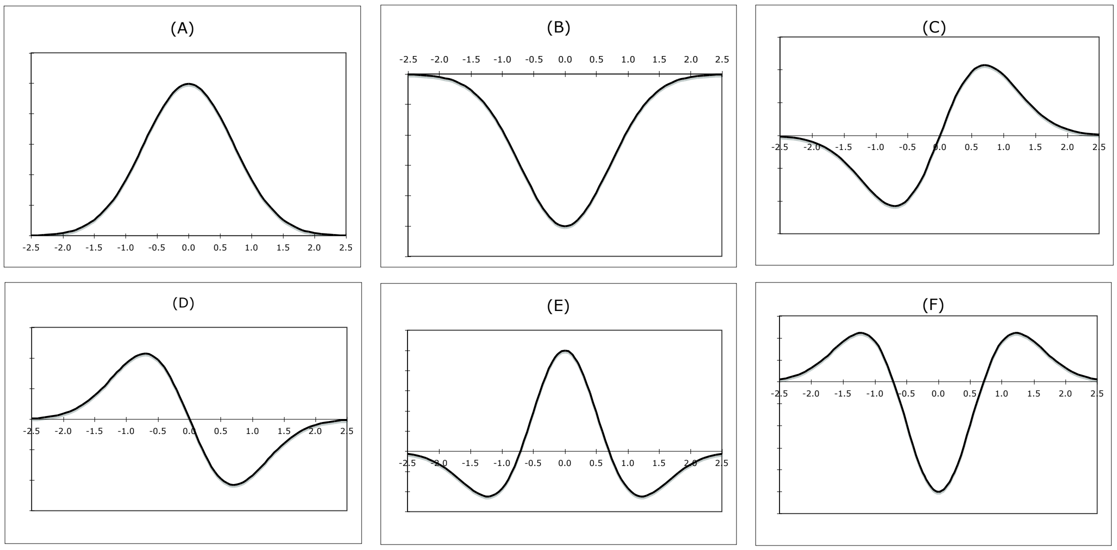
— Closed book; no notes. You may use a calculator.
— Check that you have all **13** pages (including cover page). The formula sheet is distributed separately.
— **Important note:** Except for some challenge problems, each part of this exam is designed to be answered without the answers of previous parts. The parts within a given problem tend to become more and more difficult. If you get stuck on one part, skip to the next problem and come back later if you have more time.

Problem	Score	Grader
1. (16 pts)		
2. (20 pts)		
3. (19 pts)		
4. (20 pts)		
5. (25 pts)		
Total (100 pts)		

Contents

1 A Traveling Pulse [16 points]

A pulse travels to the right in the positive $x-$direction along a stretched string. In the figure below, (A) represents a snapshot picture of this pulse, $y(x, t_0)$, taken at time $t = t_0$. The units for x on the horizontal axis are the same in all pictures. The vertical axes have not been given numerical values; you may assume that their units are arbitrary.



1.1 (4 points)

Which graph represents the transverse velocity of the pulse, $\partial y(x,t)/\partial t|_{t=t_0}$?

1.2 (4 points)

Challenge:

Which graph represents the transverse acceleration of the pulse, $\partial^2 y(x,t)/\partial t^2|_{t=t_0}$?

1.3 (4 points)

A long time later, at $t = t_1$, the pulse has been reflected at a *free* boundary and is passing the same location again but moving to the left in the negative x-direction.

Which graph represents a snapshot of the reflected returning pulse at $t = t_1$, $y(x, t_1)$?

1.4 (4 points)

Which graph represents the transverse velocity of the reflected pulse, $\partial y(x,t)/\partial t|_{t=t_1}$?

PLEASE TURN PAGE

(Intentionally blank for workspace.)

2 Explicit expression for a reflected pulse [20 points]

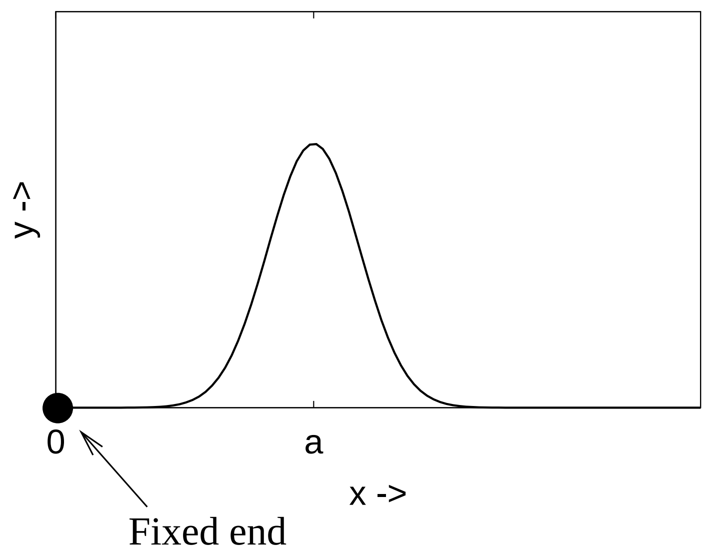
A long stretched string is oriented along the x-axis with its left end fixed at $x = 0$. The wave speed on the string is v. The solution to the wave equation for the string has the form

$$y(x,t) = f(x - vt) + g(x + vt)$$

with

$$g(x + vt) = Ae^{-(x+vt-a)^2/b^2}$$

The picture below shows the string at $t = 0$. The pulse shown is moving to the left.



2.1 (7 points)

Use the boundary condition at $x = 0$ to find an equation relating the functions $f(u)$ and $g(u)$. Do *not* insert the specific functions for f and g yet.

2.2 (7 points)

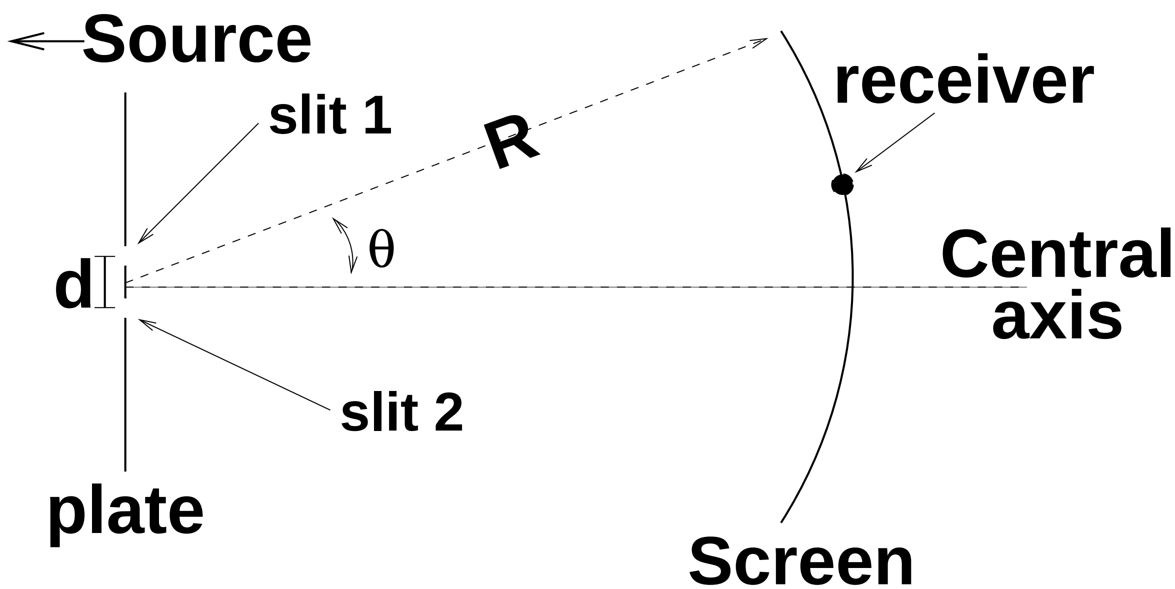
Find $f(u)$.

2.3 (6 points)

Write down $y(x,t)$ in terms of only A, v, a, b, x, t, and fundamental constants, and show that $y(x,t)$ satisfies your boundary condition from 2.1.

3 Double Slit Interference [19 points]

A beam of microwave plane waves incident from the left directly along the central axis in the figure below encounters a metal plate with one or more narrow slits. The receiver, always at distance R from the slit plate on the side opposite the generator of the beam, can be moved in an arc to vary the angle θ to the central axis. (See figure below.)



When there is only one slit open in the plate, the intensity measured on the other side of the slit by a receiver placed on the central axis of the arrangement ($\theta = 0$) at a distance R is I_0.

You now add a second slit of the same size. The two slits are $d = 5.0$ cm apart from each other, arranged symmetrically about the central axis, as shown, with $d << R$.

3.1 (5 points)

In terms of I_0, what is the intensity measured by the receiver when it is placed on the central axis ($\theta = 0$) with both slits open?

3.2 (8 points)

You move your receiver to $\theta = 15°$ and find that this is the *first* angle at which the intensity decreases to zero. What is the wavelength, λ, of the microwaves?

3.3 (6 points)

You move the detector back to the central axis ($\theta = 0$) and you cover one of the two slits with a piece of polyethylene of thickness b and index of refraction $n = 1.6$. In terms of n, b, λ, and mathematical constants as needed, what is the difference in phase $\Delta\phi$ between the microwaves arriving at the detector from the two slits?

4 Standing electromagnetic waves [20 points]

The electric field of an electromagnetic standing wave of angular frequency ω in vacuum is given by

$$\vec{E}(x,t) = E_0 \sin(kx)\cos(\omega t)\ \hat{y}$$

4.1 (3 points)

Does $\vec{E}(x,t)$ have a node or an antinode at $x = 0$?

4.2 (7 points)

$\vec{E}(x,t)$ can be written in the form of the general solution of the wave equation,

$$\vec{E}(x,t) = \vec{E}_1(x,t) + \vec{E}_2(x,t),$$

where $\vec{E}_1(x,t) \equiv f(x-ct)\hat{y}$ and $\vec{E}_2(x,t) \equiv g(x+ct)\hat{y}$. Give explicit formulas for $\vec{E}_1(x,t)$ and $\vec{E}_2(x,t)$ in terms of *only* E_0, c, k, ω, and appropriate unit vectors as needed.

Hint: $\cos(\alpha)\sin(\beta) = (\sin(\alpha+\beta) - \sin(\alpha-\beta))/2$.

4.3 (6 points)

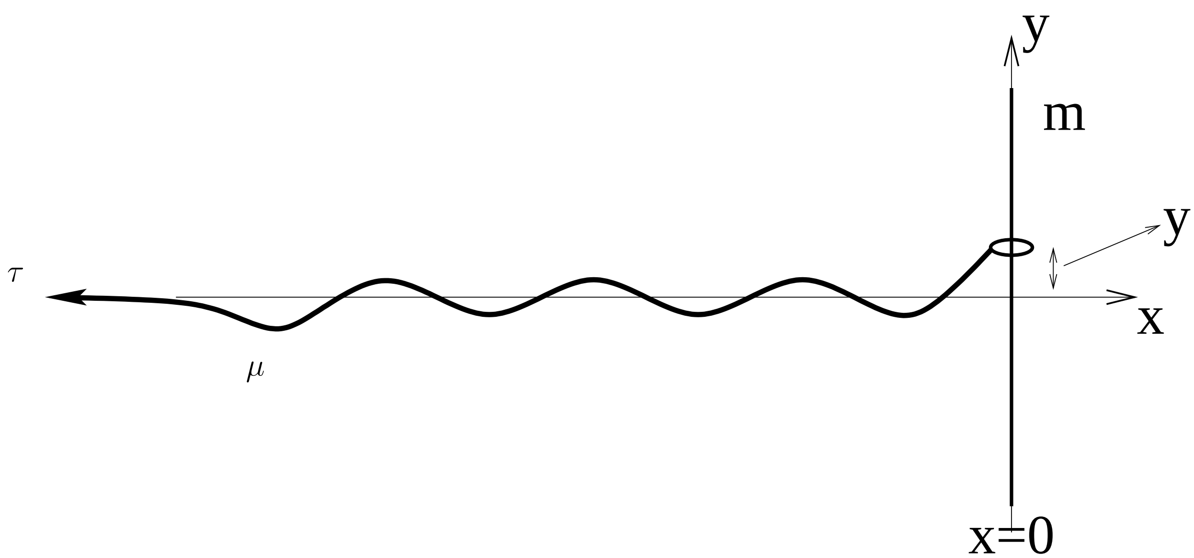
Find explicit formulas for the magnetic fields $\vec{B}_1(x,t)$ and $\vec{B}_2(x,t)$ associated with the electric fields $E_1(x,t)$ and $E_2(x,t)$ in terms of *only* E_0, c, k, ω, and appropriate unit vectors as needed.

4.4 (4 points)

What is the value at $x = 0$ of the magnetic field associated with the standing wave, $\vec{B}(0,t)$? Also, the magnetic field has either a node or an antinode at this point. Which is it?

5 The return of the ring [25 points]

A sinusoidal wave of complex amplitude $\underline{A}$ is sent from left to right along a long string oriented in the x-direction. The speed of transverse waves all along this string is c, the tension is τ. At $x = 0$ the string is attached to a vertical fixed rod by means of a frictionless ring of mass m. You may ignore the effects of gravity on the mass and on the string.



A solution to the wave equation for the string is given by

$$y(x \le 0, t) = \Re\left[\underline{A}e^{-i\omega t}(e^{ikx} + \underline{B}e^{-ikx})\right]$$

5.1 (10 points)

Fill each box with the letter corresponding to the meaning of each of the following terms from the above expression.

$\underline{A}$:

$e^{-i\omega t}$:

e^{ikx}:

$\underline{B}$:

e^{-ikx}:

(a) Radius of curvature

(b) Reflection coefficient

(c) Propagation phase of incident wave

(d) Transmission coefficient

(e) Propagation phase of reflected wave

(f) Complex amplitude of incident wave

(g) Vector potential

(i) Simple harmonic motion of each chunk at the same frequency

5.2 (8 points)

The boundary condition for the string at $x = 0$ is given by the equation of motion of the ring

$$m\frac{\partial^2 y}{\partial t^2}|_{x=0} = -\tau\frac{\partial y}{\partial x}|_{x=0}$$

Using this boundary condition, find the complex coefficient $\underline{B}$ in terms of *only* τ, m, k, c, and any numerical constants as needed.

5.3 (4 points)

What is $\underline{B}$ in the limit of $m \to \infty$ and $m \to 0$, respectively? What kind of termination of the string do these limits represent? **Hint:** Even if you have not completed 5.2, you may be able to answer this and the next problem based on physical intuition.

5.4 (3 points)

What is the numerical value of $|\underline{B}|$? Explain your result (one or two sentences) in terms of conservation of energy.

END OF EXAM