

Waves, Optics, and Particles, Fall 2004

Homework Assignment # 11

(Due Tuesday, November 23 at 5:00pm *sharp.*)

Skills to be mastered:

- Understand the wave nature of particles, be able to compute de Broglie wavelength for a particle of known energy, momentum or velocity
- Use the particle-wave correspondence to predict the results of simple scattering and “diffraction” experiments with electrons

Lectures and Readings:

- Lec 25, 11/23 (Tue): Heisenberg Uncertainty Principle.
Readings: YF 39.3; LN “Quantum I: “The Experimental Observation”; LN “Quantum II: Notes on the Heisenberg Uncertainty Principle and its applications.”
- Lec 26, 11/30 (Tue): Particles in a box; Schrödinger’s equation.
Readings: YF 39.5, 40.1–40.3; LN “Quantum III: Particle in a box . . . ,” Secs. 1.1–1.6.
- Lec 27, 12/02 (Thu): Three Nobel Ideas.
Readings: LN “Feynman Diagrams . . . ,” Secs. 3.4, 3.5, 4, 5.

1 Single-slit diffraction using a baseball

A baseball of mass 0.145 kg is used in a single-slit experiment. The “slit” is a doorway 1 m wide. The baseball is thrown through the doorway at 40 m/s. Estimate the angular half-width of the central diffraction maximum (the angle θ_1 shown in YF Fig. 39.6).

2 Single-slit diffraction using electrons

Do YF Problem 39.38.

3 Multiple slits with electrons

A system of several identical parallel narrow slits is fabricated at the Cornell Nanofabrication Facility. The system is illuminated with 1 keV electrons (that is, with electrons whose kinetic energy is 1 keV). The interference pattern shown in Fig. 1 is observed on a distant screen. The graph shows the intensity (the number of electrons striking the screen per unit area) as a function of θ .

- How many slits are there? Explain.
- What is the separation d between the slits?
- Estimate the width a of each slit.

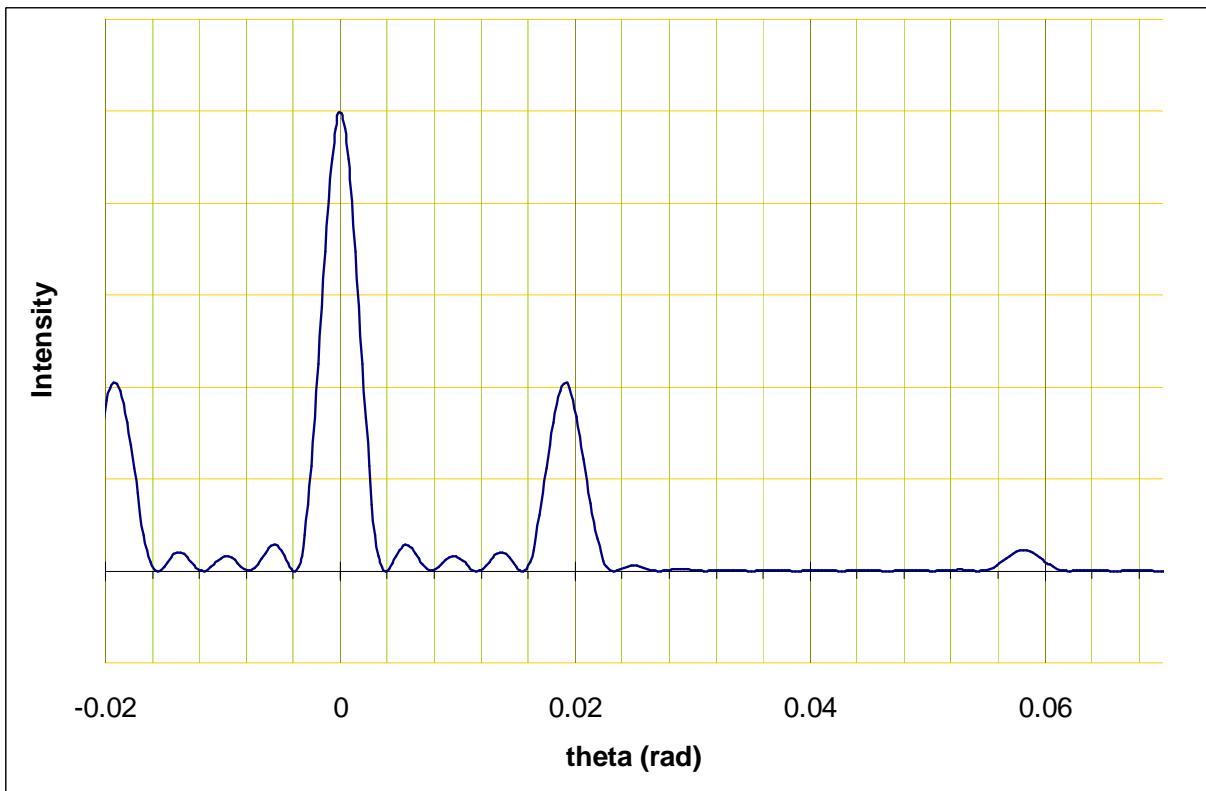


Figure 1: Interference pattern observed on a distant screen.

4 Size of the hydrogen atom

Consider a hydrogen atom. By the Heisenberg Uncertainty Principle (HUP), we know that the electron can't be located exactly at the nucleus. Suppose we know that it is typically a distance r from the nucleus.

- From the expression for the electric potential energy between two charges, what then is the typical potential energy in terms of just the elementary charge e , the Coulomb constant k , and r ?
- Assuming that the electron is confined to a region whose size is of order r , what is the uncertainty in its momentum in terms of \hbar and r ?
- Given that the *average* momentum of the electron should be zero (there is no reason for it to be traveling in one direction or another on average), what is the typical kinetic energy of the electron in terms of just \hbar , r and the mass of the electron m ?
- The electron in the atom will eventually find a probability distribution with a spread r that minimizes the total energy (KE + PE). What value for r (in Angstroms) gives the minimum energy? Is this a reasonable estimate for the size of a hydrogen atom?
- What value do you get for the minimum energy (in electron-volts)? How does this compare with the ionization energy of the hydrogen atom?