HOMEWORK ASSIGNMENT # 11

SOLUTIONS

① Single-slit diffraction using a baseball :

Mass of the baseball = $m = 0.145$ Kg

Size (width) of the slit (doorway) = $a = 1$ m

Speed of the baseball = $v = 40$ m/s

deBroglie wavelength associated with the baseball

$$= \lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34} \text{ J.s}}{0.145 \text{ Kg} \times 40 \text{ m/s}}$$

$$= 1.142 \times 10^{-34} \text{ m}$$

Angular half-width of the central diffraction maximum

$= \theta_1$

This angle, θ_1, satisfies the minimum condition :

$$\sin\theta_1 = \frac{\lambda}{a} = \frac{1.142 \times 10^{-34} \text{ m}}{1.0 \text{ m}} = 1.142 \times 10^{-34}$$

For such a small value of the sine function,

$$\theta_1 \simeq \sin\theta_1 = 1.142 \times 10^{-34} \text{ radians}.$$

(2) Single-slit diffraction using electrons :

Mass of electron $= m = 9.11 \times 10^{-31}$ Kg

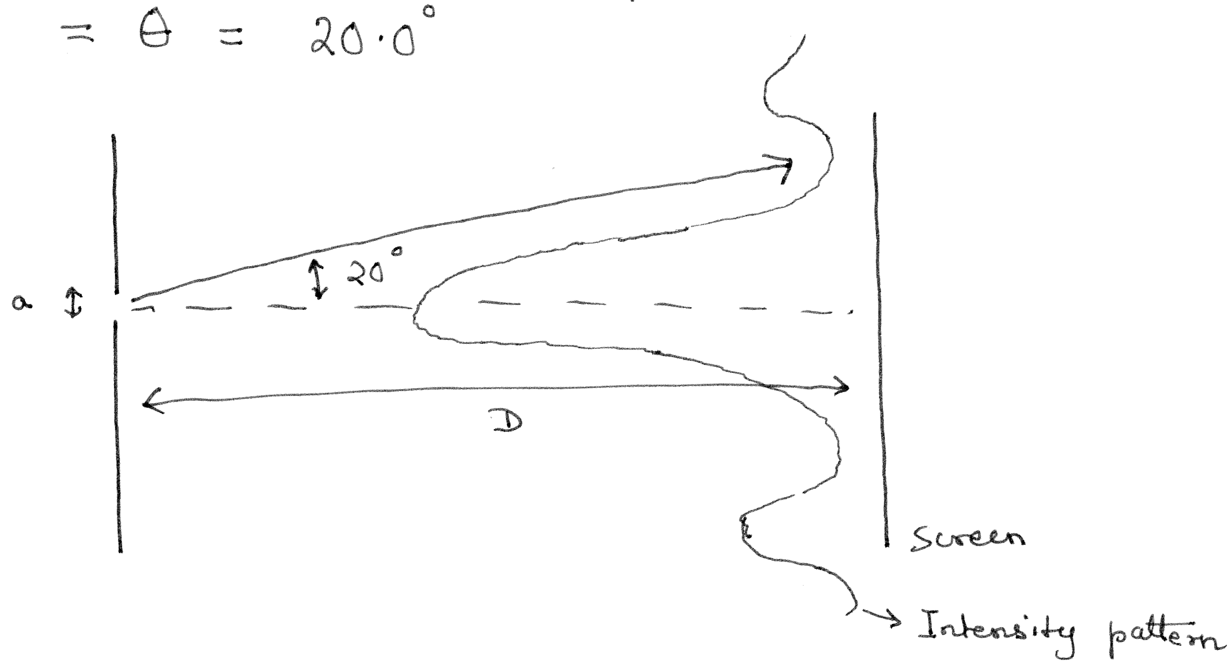
(a) Slit width $= a = 150$ nm $= 1.5 \times 10^{-7}$ m

Distance to the screen $= D = 24.0$ cm

$= 0.24$ m

Angular position of the first minimum

$= \theta = 20.0^\circ$



To find the velocity of the electrons, v, note that the de Broglie wavelength is given by $\lambda = \frac{h}{mv}$.

Further, λ is related to the first minimum by the condition : $\sin\theta = \frac{\lambda}{a}$

Therefore, $v = \frac{h}{m\lambda} = \frac{h}{m a \sin\theta}$

$$= \frac{6.626 \times 10^{-34} \text{ Js}}{9.11 \times 10^{-31} \text{ Kg} \times 1.5 \times 10^{-7} \text{ m} \times \sin(20^\circ)}$$

$$\boxed{= 1.4177 \times 10^{4} \text{ m/s}}$$

(b) Next larger angle at which no electrons hit the screen corresponds to the angular position of the next minimum (the second minimum). The corresponding condition is :

$$a \sin\theta = 2\lambda$$

$$\sin\theta = \frac{2\lambda}{a} = 2 \sin(20^\circ)$$

where we use the fact that the first minimum occurs at 20°, and, therefore, $\sin(20^\circ) = \frac{\lambda}{a}$.

Hence, $\theta = \sin^{-1}(2 \sin 20^\circ) = \underline{43.16^\circ}$

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(3) Multiple slits with electrons :

(a) No. of minima between two primary maxima = 4

$\therefore$ No. of slits = $N = 4 + 1 = 5$

(we use the fact that the no. of minima between two primary adjacent maxima is $N-1$ for N-slit interference)

(b) Position of the first minimum = $\theta \simeq \frac{0.02}{5}$ radians.

At this position the following condition must be satisfied : $d \sin\theta = \frac{1}{5} \times \lambda$

where d = slit separation

λ = de Broglie wavelength of electrons

$$= \frac{h}{\sqrt{2mE}}$$

$E \rightarrow$ Kinetic energy of electron = 1keV = 1.6×10^{-16} J

$m \rightarrow$ mass of electron = 9.11×10^{-31} kg

$$= \frac{6.626\times10^{-34}\ \text{Js}}{\sqrt{2\times9.11\times10^{-31}\ \text{kg}\times1.6\times10^{-16}\ \text{J}}}$$

$$= 3.88\times10^{-11}\ \text{m}$$

$$\therefore\ d = \frac{\lambda}{5\sin\theta} \approx \frac{\lambda}{5\,\theta}$$ (small angle approximation)

$$= \frac{3.88\times10^{-11}\ \text{m}}{5\times\frac{0.02}{5}}$$

$$= \underline{1.94\times10^{-9}\ \text{m} = 1.94\ \text{nm}}$$

(c) Notice that there is a missing maximum at around the angular position, $\phi \simeq 0.04$ radians. This should correspond to the first minimum due to diffraction.

So, $a\sin\phi = \lambda$

$$\Rightarrow\ \sin\phi \approx \phi = \frac{\lambda}{a} = \frac{3.88\times10^{-11}\ \text{m}}{a}$$

So $$a = \frac{3.88\times10^{-11}\ \text{m}}{0.04} = \underline{9.7\times10^{-10}\ \text{m}}$$

$$= \underline{0.97\ \text{nm}}$$