1 ⓐ $x - y = (-5 + 12i) - (4 - 3i) = -5 - 4 + 12i + 3i$

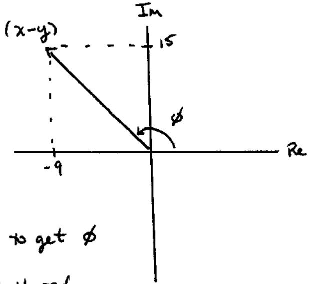
$= \boxed{-9 + 15i}$

Now we want to express this in polar form $Ae^{i\phi}$

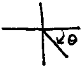
$A = \sqrt{(\text{real part})^2 + (\text{imaginary part})^2}$

$= \sqrt{(-9)^2 + (15)^2}$

$\cong 17.5$



Let $\theta = \tan^{-1}\left(\frac{15}{-9}\right) \cong -1.03$ rad

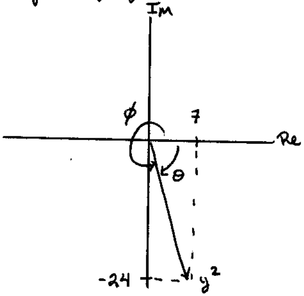
this looks like: 

so we have to add π rad to θ to get ϕ

$\Rightarrow \phi = \pi + \theta = \pi - 1.03 \cong 2.11$ rad

Finally, we get: $-9 + 15i = \boxed{17.5\, e^{i(2.11)}}$

ⓑ $y^2 = y \cdot y = (4 - 3i)(4 - 3i) = 16 + 9i^2 - 12i - 12i = \boxed{7 - 24i}$



$A = \sqrt{7^2 + (-24)^2} = \sqrt{625} = 25$

$\theta = \tan^{-1}\left(\frac{-24}{7}\right) \cong -1.3$ rad

θ is in the correct quadrant, but we want an angle that's greater than ~~zero~~. So, let's add 2π

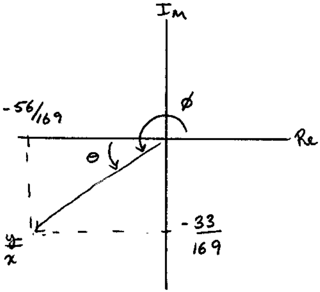
$$\phi = 2\pi + \theta = 2\pi - 1.3 = 4.9 \text{ rad}$$

$$\Rightarrow 7 - 24i = \boxed{25\, e^{i(4.9)}}$$

ⓒ $\dfrac{y}{x} = \dfrac{4-3i}{-5+12i}$ To obtain standard form, multiply top and bottom by the complex conjugate of the denominator.

$$= \frac{4-3i}{-5+12i} \cdot \frac{-5-12i}{-5-12i} = \frac{-20-48i+15i+36i^2}{25-60i+60i-144i^2}$$

$$= \frac{-56-33i}{25+144} = \boxed{\frac{-56}{169} - \frac{33}{169}i}$$



$$A = \sqrt{\left(\frac{-56}{169}\right)^2 + \left(\frac{-33}{169}\right)^2} = \frac{65}{169}$$

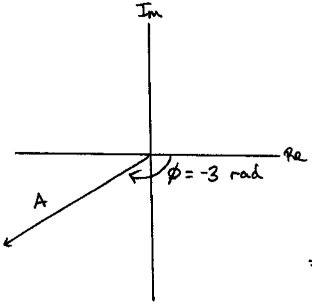
$$\theta = \tan^{-1}\left(\frac{-33/169}{-56/169}\right) \doteq 0.53 \text{ rad}$$

$$\phi = \pi + 0.53 \doteq 3.7 \text{ rad}$$

$$\Rightarrow \frac{-56}{169} - \frac{33}{169}i = \boxed{\frac{65}{169}\, e^{i(3.7)}}$$

ⓓ $e^{y} = e^{4-3i} = \boxed{e^{4}\, e^{-3i}}$ Here, $A = e^{4}$

$\phi = -3$ rad



We can use Euler's equation to find the Cartesian form.

$$e^{i\phi} = \cos\phi + i\sin\phi$$

$$\Rightarrow e^{4}e^{-3i} = e^{4}\left[\cos(-3) + i\sin(-3)\right]$$

$$= e^{4}\left[-0.99 + i(-0.14)\right]$$

$$= \boxed{-54 - 7.6i}$$

ⓔ $|x| = |-5+12i| = \sqrt{(-5)^2 + (12)^2} = \sqrt{169} = \boxed{13}$

To find the polar form, note that $A = 13$

$\phi = 0$ rad

$$\Rightarrow 13 = \boxed{13\,e^{i0}}$$

ⓕ $yy^* = (4-3i)(4+3i) = 16 - 9i^2 - 12i + 12i = 16 + 9$

$$= \boxed{25} = \boxed{25\,e^{i0}}$$

2 (2.1) $x(t) = -2.0\text{ cm} + (1.5\text{ cm})\cos\left[\left(314\ \frac{\text{rad}}{\text{sec}}\right)t + \left(\frac{2\pi}{3}\text{rad}\right)\right]$

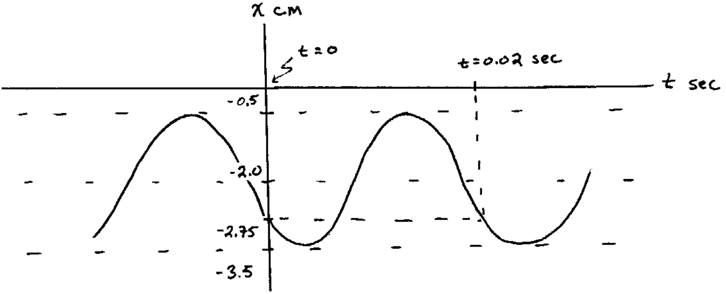
$$\underset{x_{eq}}{\uparrow} \qquad \underset{A}{\uparrow} \qquad \underset{\omega}{\uparrow} \qquad \underset{\phi_0}{\uparrow}$$

Let's find the x-intercept:

$$\text{at } t=0, \quad x(0) = -2 + 1.5\cos\left(\frac{2\pi}{3}\right) = -2.75 \text{ cm}$$

Also, we know: $f = \frac{\omega}{2\pi} \doteq 50$ Hz

$$T = \frac{1}{f} = 0.02 \text{ sec}$$



(2.2) From figure 1:

$x_{eq} \doteq -4$ mm

$A \doteq 6$ mm

$T \doteq 1.5$ sec

$f = \frac{1}{T} = \frac{2}{3}$ Hz

$$\omega_0 = \frac{2\pi \text{ rad}}{T \text{ sec}} = 4.2 \frac{\text{rad}}{\text{sec}}$$

To find ϕ_0, look at x at $t=0$.

$$x(t=0) = 1\,mm = -4\,mm + 6\,mm \cos(\phi_0)$$

$$\frac{5\,mm}{6\,mm} = \cos(\phi_0)$$

$$\phi_0 = \cos^{-1}\left(\frac{5}{6}\right)$$

$$\phi_0 = 0.59 \text{ rad or } -0.59 \text{ rad}$$

note that -0.59 rad $= 5.7$ rad

So, we need to check both answers for ϕ_0

$\underline{\phi_0 = 0.59}$: From Figure 1, we know that $\frac{dx}{dt}$ is positive at $t=0$

$$\left.\frac{dx(t)}{dt}\right|_{t=0} = -6 \sin \phi_0 \quad \text{now plug in } \phi_0 = 0.59 \text{ rad}$$

$$= -6 \sin(0.59)$$ uh oh! this isn't positive! reject this value for ϕ_0.

$\underline{\phi_0 = 5.7}$: plug in $\phi_0 = 5.7$ rad

$$-6 \sin \phi_0 = -6 \underbrace{\sin(5.7)}_{\text{this is negative}}$$

$$= \text{a positive value}$$ ok value for ϕ_0.

$$\Rightarrow \boxed{\phi_0 = 5.7 \text{ rad}}$$