Cornell University

Department of Physics

Phys 214 September 2, 2004

Waves, Optics, and Particles, Fall 2004

Homework Assignment # 2

(Due Thursday, September 9 at 5:00pm *sharp.*)

Agenda and readings for the week of September 6:

Skills to be mastered:

- differentiating between the Equation of Motion and a solution of it ;
- verifying a general solution;
- finding particular solutions given initial conditions;
- determining the complex amplitude from the initial conditions for a simple harmonic oscillator;
- determining the (real) amplitude and the initial phase from the complex amplitude;
- using the complex representation to solve differential equations.

Lectures and Readings:

Readings marked LN are from the course lecture notes to be found at http://people.ccmr.cornell.edu/~muchomas/P214.
Readings marked VW are from *Vibrations and Waves* by A. P. French.
Readings marked AG are from the draft text by Alan Giambattista, available online at http://www.physics.cornell.edu/p21
Readings marked YF are from the text Young and Freedman, *University Physics*, 11th edition.

- Lec 4, 09/07 (Tue): Damped, driven oscillator; resonance.
  Readings: LN "Simple Harmonic Motion," Sec. 6; VW pp. 62-68 and 77-92); AG *Oscillations* Sections 24.6 and 24.7.
- Lec 5, 09/09 (Thu): Wave equation for the string; standing waves.
  Readings: LN "Intro to Waves: Waves on a String and Standing Waves," Sec. 1-4.3.1; VW pp. 161-167; AG *Waves* Sections 25.4 and 25.5; AG *Superposition* Sections 26.3 and 26.4.

1 Identifying solutions and general solutions

The equation of motion (E of M) for an ideal mass-spring system is $-k(x - x_{\mathrm{eq}}) = m\frac{d^2x}{dt^2}$. For each of the following expressions, state whether it could represent a *general* solution to the E of M, or whether it could be a solution but not a general solution, or whether it cannot be a solution. In a quick phrase or two, explain under what condition(s) the expression could be a solution (answer with a phrase like "Yes, a general solution if ω_1=...") or explain why it cannot be. Also identify the adjustable parameters. (All quantities other than x and t are constants. Assume all constants other than i to be real numbers.)

(a) $x(t) = C_1 + C_2 \cos\omega_1 t$;

(b) $x(t) = x_{\rm eq} + A_1 \sin\omega_1 t + A_2 \cos\omega_1 t$;

(c) $x(t) = x_{\rm eq} + A_1 \sin\omega_1 t + A_2 \sin\omega_2 t$;

(d) $x(t) = x_{\rm eq} + \Re(A_1 e^{i\omega_0 t} - A_2 e^{-i\omega_0 t})$, where $\Re$ stands for *the real part of* ;

(e) $x(t) = x_{\rm eq} + A \cos[\omega_0(t - t_0)]$;

(f) $x(t) = x_{\rm eq} + A \tan\omega_0 t + B \sec\omega_0 t$;

(g) $x(t) = x_{\rm eq} - A \cos(\phi_0 - \omega_0 t)$;

2 Damped oscillator

A damped oscillator is modeled as a mass m at equilibrium point $x = x_{\rm eq}$ acted on by (1) an ideal spring of spring constant k and (2) a viscous drag force proportional to the velocity: $\vec{F}_{\rm drag} = -bm\vec{v}$.

(a) Derive the equation of motion.
Hint: You can check (especially your signs) against Eq. (24-21) of AG. Be careful in comparing, though, because AG's "b" is actually our "$b \cdot m$" and thus the equations won't look *exactly* the same.

(b) *Use the complex representation* to find a *real general solution* to the equation of motion. For this problem, you may *assume* that the drag constant b is relatively small: $b < 2\sqrt{k/m}$. What are the adjustable parameters?

Hint: To make your general solution, assume that $x(t)$ has the form $x = \underline{A}e^{\underline{\alpha}t}$ where $\underline{\alpha}$ is complex. Be sure to take the real part to get your answer. Compare your answer with the general solution in AG Eq. (24-23).

(c) Find an expression for and sketch a graph of the particular solution, if $k = 632$ N/m, $m = 1$ kg, $b = 0.5$ s^{-1}, $x_{\rm eq} = 0$, $x_0 = 5$ mm, and $v_0 = 0$.

3 Shake it up!

Equipment to be used in airplanes or spacecraft is often subjected to a shake test to be sure it can withstand the vibrations that may be encountered during flight. A radio receiver of mass 5.24 kg is set on a platform that vibrates in SHM at 120 Hz. The maximum acceleration of the radio during the vibrations is 98 m/s^2.

(a) Find the amplitude and maximum speed of the radio's motion.

(b) Suppose that at $t = 0$ the radio is at $x = x_{\rm eq} + 0.12$ mm and is moving in the $-x$ direction. What is the complex amplitude of the radio's motion?

4 An unusual oscillator

An object of mass m is attached to an ideal spring (spring constant k). The oscillator is driven by an applied force $F(t) = F_0 \cos\omega t$. An unusual drag force is exerted on the object; the drag force is given by $f = -C\frac{d^3x}{dt^3}$ where C is a constant. Choose your origin so that $x_{eq} = 0$.

(a) Find the equation of motion for this driven oscillator.

(b) After all transients have died out, the object oscillates at the same frequency as the driving force (ω). Write $x(t)$ for the steady-state oscillations as $x(t) = \underline{A}e^{i\omega t}$. Substitute this form for $x(t)$ into the equation of motion and solve for the complex amplitude $\underline{A}$. (It is not necessary to put $\underline{A}$ into standard Cartesian or polar form.)