Cornell University

Department of Physics

Phys 214 September 9, 2004

Waves, Optics, and Particles, Fall 2004

Homework Assignment # 3

(Due Thursday, September 16 at 5:00pm *sharp.*)

Agenda and readings for the week of September 13:

Skills to be mastered:

- be able to use complex representation to solve differential equations;
- be able to convert between wavelength (λ), wavenumber or spatial frequency ($\kappa = 1/\lambda$), and wavevector ($k = 2\pi/\lambda$);
- given the wave speed (v) and any one of the quantities (T, f, ω), be able to find any one of (λ, κ, k), and vice versa;
- given any one of the quantities (T, f, ω) AND any one of the quantities (λ, κ, k), be able to find v;

Lectures and Readings:

LN = course lecture notes at http://people.ccmr.cornell.edu/~muchomas/P214.
VW = *Vibrations and Waves* by A. P. French.
AG = text by Alan Giambattista available at http://www.physics.cornell.edu/p214/readings.html.
YF = Young and Freedman, *University Physics*, 11th edition.

- Lec 4, 09/14 (Tue): Normal modes; boundary conditions for string; dispersion relation $v = \sqrt{\tau/\mu}$.
Readings: LN "Intro to Waves: Waves on a String and Standing Waves," Sec. 4.3–5; VW pp. 161-170; AG 26.3 and 26.4.
- Lec 5, 09/16 (Thu): Wave equation for sound.
Readings: LN " Other Waves: Sound," Sec. 1–2; VW pp. 170-175; AG 25.6 and 25.7.

1 Resonance for an unusual oscillator

In PS 2 Problem 4, you found the complex amplitude for an unusual oscillator as a function of ω, the frequency of the driving force:

$$\underline{A} = \frac{F_0}{k - m\omega^2 - iC\omega^3} = \frac{F_0}{m({\omega_0}^2 - \omega^2) - iC\omega^3} \tag{1}$$

where $\omega_0 \equiv \sqrt{k/m}$.

(a) What is the actual (real) amplitude of the oscillations, as a function of ω? [*Hint*: Think carefully about whether you want to take the real part of $\underline{A}$ or the magnitude of $\underline{A}$.]

(b) Sketch a graph of $A(\omega)$ for $F_0 = 1$ N, $m = 1$ kg, $\omega_0 = 100$ rad/s, and $C = 10^{-3}$ kg·s. Does the system exhibit resonance? Explain in a sentence or two.

2 Seismograph

Do problem 4-6 in VW p. 113. [Note that their γ is our bm; $\omega_0 \equiv \sqrt{k/m}$; and Q is defined on VW p. 67, Eq. (3-37).]

3 Energy in a driven oscillator

An oscillator (k, m) with natural frequency $\omega_0 = \sqrt{k/m}$ is driven by a driving force $F(t) = F_0 \cos \omega t$. The damping force on the oscillator is $f = -bmv$. In LN, we find that the steady-state oscillations are of the form $x(t) = A\cos(\omega t + \phi_0)$, where $A = \frac{F_0/m}{\sqrt{(\omega^2-{\omega_0}^2)^2+(b\omega)^2}}$ and $\phi_0 = -\arctan \frac{b\omega}{{\omega_0}^2-\omega^2}$. (You may leave A and ϕ_0 in your answers, when possible, rather than substituting these expressions.)

(a) Find the instantaneous rate at which the damping force dissipates energy. [Hint: Use $P = \vec{F} \cdot \vec{v}$.]

(b) What is the average rate at which the damping force dissipates energy during one cycle? [Hint: since $\sin^2(\omega t + \phi_0) + \cos^2(\omega t + \phi_0) = 1$, what must be the average value of $\sin^2$ during one cycle?]

(c) Find the instantaneous rate at which the driving force puts energy into the oscillator.

(d) What is the average rate at which the driving force puts energy into the oscillator during one cycle? [Hint: expand $\sin(\omega t + \phi_0)$ using a trig identity and then average each term separately. It may also help to use a trig identity for $\sin \omega t \cos \omega t$.]

(e) How do your answers to (b) and (d) compare? Explain.

4 Conversion of wave quantities

(a) A fisherman notices a buoy bobbing up and down in the water in ripples produced by waves from a passing speedboat. These waves travel at 2.5 m/s and have a wavelength of 7.5 m. (a) At what frequency does the buoy bob up and down? (b) Find the wavenumber κ, the wavevector k, and the angular frequency ω for these waves. [Note: see LN for definitions of wavenumber and wavevector. The terminology differs from book to book; e.g. the magnitude of the wavevector is often called the wavenumber!]

(b) The frequency range of sound waves that can be heard by a person with excellent hearing is 20 Hz – 20,000 Hz. What is the range of wavelengths of these waves in air at 20 degrees Celsius?

5 Plucking a harpsichord string

A 1.8-m-long harpsichord string is plucked at a point 20.0 cm from one end. The plucking point is raised 4.0 mm above its equilibrium position and then released.

(a) What are the slopes of the string, on either side of the plucking point, as it is plucked?

(b) Consider the model from lecture and the lecture notes of a string with one end fixed and the other passing through a hole with a fixed applied tension. In this model the string doesn't stretch but the total length of string between the boundaries changes. For the configuration described in (a), how much has the chunk that started at the position of the plectrum moved in the *horizontal* direction? (See Figure 1.)

(c) Would you be justified in working in the small amplitude approximation in this case? In your answer, refer to the slopes calculated in (a) and compare the horizontal motion of the chunk in (b) to its veritical motion.

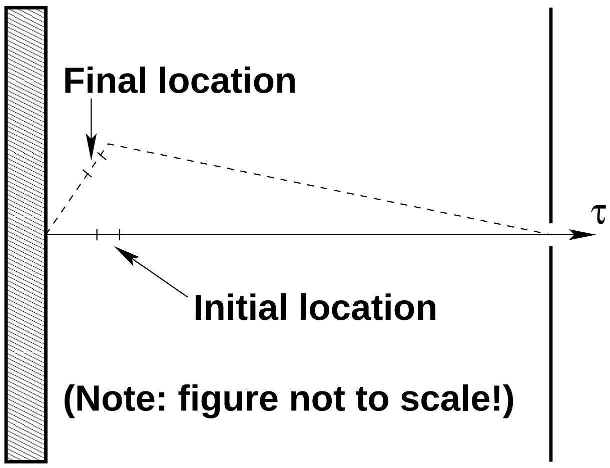

Figure 1: A small chunk of string at the position of the plectrum moves both vertically and horizontally