Cornell University

Department of Physics

Phys 214 October 7, 2004

Waves, Optics, and Particles, Fall 2004

Homework Assignment # 6

(Due Thursday, October 14 at 5:00pm *sharp*.)

Prelim 1 Information
Solutions available at: `http://www.physics.cornell.edu/p214/p1solutions.pdf`
Histogram available at: `http://www.physics.cornell.edu/p214/p1histogram.pdf`

Reminder: Lab Experiment 2 begins on the day we return from fall break!

Agenda and readings for the weeks of October 4 and October 11:

Skills to be mastered:

- understanding and being able to show that $\vec{E} \perp \vec{B}$ for electric and magnetic fields in the absence of sources (i.e., free charges or currents);
- given the direction of propagation of a an EM wave, being able to obtain the function $\vec{B}(\vec{r},t)$ from $\vec{E}(\vec{r},t)$, and vice versa;
- being able to determine the allowed modes for EM standing waves for different types of boundary conditions and to locate the nodal planes of $\vec{E}$ and $\vec{B}$;
- given the polarization of $\vec{E}(\vec{r},t)$, being able to find the polarization of $\vec{B}(\vec{r},t)$, and vice versa;
- be able to identify traveling wave solutions $f(x-ct)$ and $g(x+ct)$ and pick out the velocity;
- be able to use the general solution $f(x-ct)+g(x+ct)$ to the wave equation to find particular solutions given the initial conditions (i.e., the string shape and velocity distribution at $t=0$);
- understanding superposition of *both* displacements and velocities for combinations of left and right pulses.

Lectures and Readings:

- Lec 12 & 13, 10/05 (Tue) & 10/07 (Thu): E&M Pol Rule III, Chunk velocities, Superposition, General solution to the wave equation.
Readings: LN "Wave Phenomena I," Sec. 4; VW pp. 228–230; AG sections 26.1–26.2.

- 10/12 (Tue): **Fall break!!!**
- Lec 14, 10/14 (Thu): Reflection at fixed and free boundaries.
  Readings: LN "Wave Phenomena I," Sec. 5; VW pp. 253-259; AG section 26.3.

1 Crystallography

In order to study the structure of a crystalline solid, you want to illuminate it with EM radiation whose wavelength in vacuum is comparable to the spacing of the atoms in the crystal ($\approx$ 0.20 nm). (a) What is the frequency of the EM radiation? (b) In what part of the EM spectrum (radio, visible, etc.) does it lie?

2 Applying Maxwell's equations

The magnetic field of a traveling plane EM wave in vacuum is

$$\vec{B}(\vec{r},t) = \vec{B}_0 \sin(ky - \omega t)$$

(a) Apply one of Maxwell's equations to show that $B_{0y} = 0$. Include a sketch that clearly shows the closed surface or closed loop used for the integral.

(b) Apply another one of Maxwell's equations to find a relationship between $\partial B_x/\partial t$ and $\partial E_z/\partial y$. Include a sketch that clearly shows the closed surface or closed loop used for the integral. Use this relationship to write an equation for $E_z(\vec{r},t)$ using no constants other than k, ω, and B_{0x}.

[Please note that this wave does *not* travel along the x axis. Therefore, you cannot simply use the results derived in class from Maxwell's equations for a wave traveling along the x axis.]

3 Detecting an EM wave

The magnetic field of a traveling plane EM wave in vacuum is

$$\vec{B}(\vec{r},t) = \vec{B}_0 \cos(\omega t + kz + \pi/12)$$

where $k = 3.0$ rad/m, $|\vec{B}_0| = 5.0 \times 10^{-10}$ T, and the direction of $\vec{B}_0$ is 37 degrees above the $-x$ axis (and 53 degrees from the $+y$ axis) in the xy plane.

(a) What is the direction of propagation?

(b) The electric field of the wave is $\vec{E}(\vec{r},t) = \vec{E}_0 \cos(\omega t + kz + \pi/12)$. What is the amplitude vector $\vec{E}_0$ of the electric field of the wave? Give magnitude and direction.

(c) What are the frequency f and wavelength λ?

(d) You have a coil of wire of radius 2.0 cm with 50 turns. You want to use this coil as an antenna to detect the EM wave. How would you orient the antenna for maximum sensitivity? (Give the direction of the loop axis.) What would be the amplitude of the induced emf in the antenna? [Hint: an exact calculation of the induced emf is quite messy. Instead, do an approximate calculation justified by the fact that the coil radius is small compared to the wavelength.]

(e) To increase the sensitivity of the antenna, you could use a coil with a larger radius. Is there any upper limit to how large you can make the radius and still gain sensitivity? Explain.