Cornell University

Department of Physics

Phys 214 October 7, 2004

# Waves, Optics, and Particles, Fall 2004

## Homework Assignment # 7

(Due Thursday, October 21 at 5:00pm *sharp.*)

## Agenda and readings for the week of October 18:

Skills to be mastered:

- identifying traveling wave solutions $f(x-ct)$ and $g(x+ct)$ picking out the velocity;
- understanding superposition of *both* displacements and velocities for combinations of left and right pulses.
- using the general solution $f(x-ct)+g(x+ct)$ of the wave equation to find particular solutions;
- finding resulting time history (snapshots at later times, particle histories, velocity distributions) from given initial conditions.
- finding solutions during and after reflection of pulses from fixed, free, and other boundary conditions;
- finding solutions during and after reflection and transmission of pulses from changes in medium.

Lectures and Readings:

- Lec 15, 10/19 (Tue): Complex representation of reflection/transmission, two-slit interference. **Readings: "Wave Phenomena II: Interference," Secs. 1-3.2; VW pp. 280-284.**
- Lec 16, 10/21 (Thu): Two- and $N$- slit interference pattern. **Readings: LN "Wave Phenomena II: Interference," Sec. 3.2–3.3; VW pp. 284-288.**

## 1 Superposition of position and velocities

A pulse (shown on Figure 1) is heading in the $x$ direction at 100 m/s towards the fixed end of a string (at $x = 0$).

(a) Sketch snapshots (graphs of $y$ vs. $x$ at fixed $t$) at $t = 0.035$ s and $t = 0.045$ s.

(b) Sketch the velocity distributions (graphs of $\partial y/\partial t$ vs. $x$ at fixed $t$) at $t = 0$, $t = 0.035$ s, and $t = 0.045$ s.

(c) Sketch a particle history (graphs of $y$ vs. $t$ at fixed $x$) for the endpoint ($x = 0$).

(d), (e), (f): Repeat parts (a), (b), and (c) if the end at $x = 0$ is free (in the $y$ direction) instead of fixed.

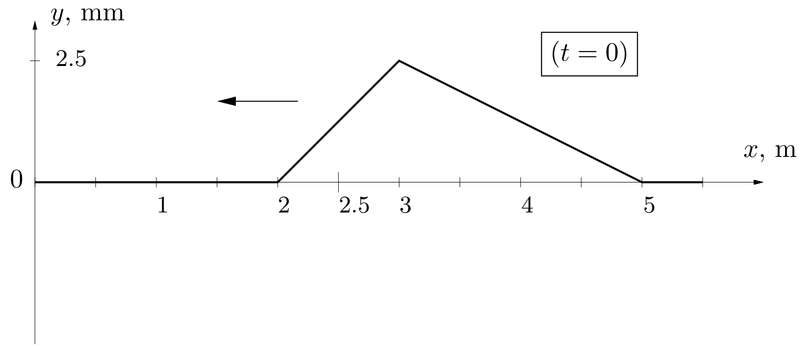


Figure 1: Traveling pulse.

# 2 Standing waves and the principle of superposition

A standing wave in a thin pipe with ends at $x = 0$ and $x = L$ is described by

$$s(x,t) = A\cos(kx)\sin(\omega t)$$

(a) Is the end at $x = 0$ open or closed? Explain. What is the boundary condition on $s(x = 0, t)$?

(b) Rewrite this standing wave in the form of the general solution

$$s(x,t) = f(x - vt) + g(x + vt)$$

by explicitly finding the functions $f(u)$ and $g(u)$. [Hint: Rewrite $\cos(kx)$ and $\sin(\omega t)$ as sums or differences of complex exponentials.]

(c) $g(x + vt)$ is a wave traveling in the $-x$ direction and $f(x - vt)$ is its reflection from the boundary at $x = 0$. Show that the relationship between the functions $f(u)$ and $g(u)$ is as expected from our discussion in class of the reflection of pulses from fixed and free ends.

# 3 Predicting the future of a plucked guitar string

At $t = 0$, a guitar string fixed at both ends ($x = 0$ and at $x = L$) is plucked at its midpoint so that its initial shape is an isosceles triangle (Fig. 2) and its initial velocity is zero everywhere. The resulting wave on the string has the general form $y(x,t) = f(x - vt) + g(x + vt)$.

(a) Use the initial condition $v_y(x, t = 0) = 0$ to show that $f(u) = g(u)$.

(b) Use the initial shape of the string to sketch the function $f(u)$ in the range $0 \le u \le L$.

(c) Apply the boundary condition at $x = 0$. What can you conclude about $f(u)$? [Hint: use a variable substitution $u = vt$.]

(d) Apply the boundary condition at $x = L$ and show that $f(L - u) = -f(L + u)$.

(e) You now have enough information to completely determine $f(u)$. Sketch $f(u)$ in the range $-2L \le u \le 4L$.

(f) Sketch the shape of the string at $t = L/4v$ where $v$ is the wave speed.

# 4 Sound reflection and transmission at an air-water boundary

Do VW Problem 8-5 part (a) on p. 298.

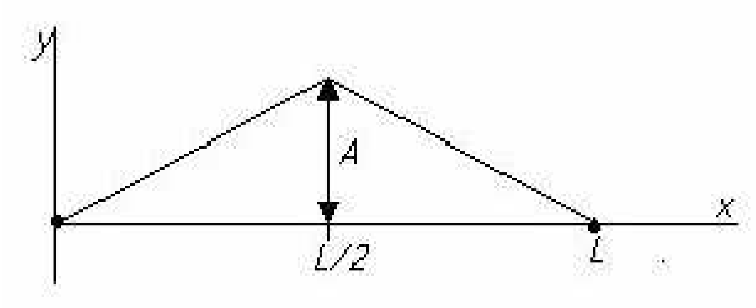


Figure 2: Initial shape of a plucked guitar string.