Cornell University

Department of Physics

Phys 214 November 5, 2004

Waves, Optics, and Particles, Fall 2004

Homework Assignment # 9

(Due Thursday, November 11 at 5:00pm *sharp*.)

Agenda and readings for the weeks of November 1 and November 8:

Skills to be mastered:

- Understand the pattern produced by N slit interference
- Understand the locations of the principal maxima, lesser maxima, and minima in an N slit pattern
- Understand the concept of *width* of a maximum in an interference pattern
- Understand the properties of the interference pattern produced by diffraction on a finite slit.

Lectures and Readings:

- Lec 19, 11/2 (Tue): Finite-slit interference
  Readings: LN "Wave Phenomena II: Interference," Sec. 4; VW pp. 288-294.
- Lec 20, 11/4 (Thu): N finite slits: combining diffraction and interference
  Readings: LN "Wave Phenomena II: Interference," Sec. 5; VW pp. 284-298; AG section 25.8. Note: for more detail about interference and diffraction, see Chapters 35 and 36 in Young and Freedman (11th edition), or the corresponding chapters in any other introductory physics text.
- Lec 21, 11/9 (Tue): Differential equation form for conservation laws; conservation of momentum.
  Readings: LN "Wave Phenomena III: Transport of momentum and energy," Secs. 1, 2, 3; VW pp. 237-244.
- Lec 22, 11/11 (Thu): Conservation of energy, power, special cases for traveling waves.
  Readings: LN "Wave Phenomena III: Transport of momentum and energy," Sec. 4.

1 N-slit interference and diffraction

A system of several identical parallel narrow slits is illuminated with red light of wavelength $\lambda = 720$ nm. The interference pattern shown in Fig. 1 is observed on a screen 4.0 m from the slits. The graph shows the intensity in mW/m^2 as a function of x, the distance on the screen from the center of the pattern.

(a) How many slits are there? Explain.

(b) What is the separation d between the slits? Compare your answer to the typical thickness of a human hair, about 0.01 cm.

(c) What is the width a of each slit?

(d) What would be the intensity at $x = 0$ if all but one of the slits were closed? Explain.

(e) What would the intensity graph as a function of x look like if the slits were very narrow ($a \ll \lambda$)? Assume the same number of slits and the same slit spacing d. Don't worry about the scale of the intensity axis on the graph; we just want a sketch that's qualitatively correct.

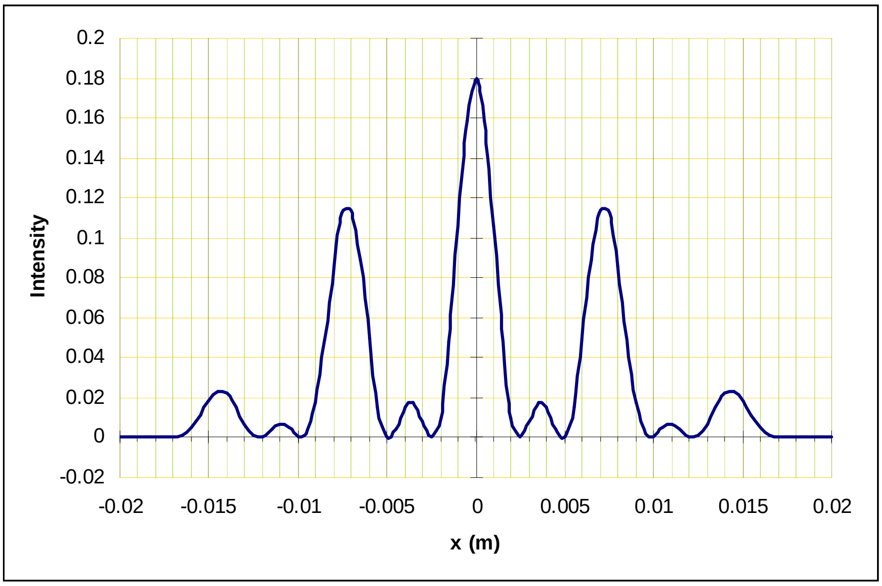

Figure 1: Interference pattern observed on a distant screen.

2 Measuring wavelengths with a grating

A grating is used to analyze a light source. The screen-to-grating distance is 50.0 cm and the grating has 5000.0 slits/cm. Spectral lines are observed at the following angles: 12.98°, 19.0°, 26.7°, 40.6°, 42.4°, 63.9°, and 77.6°.

(a) How many different wavelengths are present in the spectrum of this light source? Find each of the wavelengths.

(b) If a different grating with 2000.0 slits/cm were used, how many spectral lines would be seen on the screen on *each side* of the central maximum? Explain.

3 Resolving Power of a Grating

A grating with N slits is illuminated with a mixture of light with two wavelengths, λ and λ'. The wavelengths are very nearly equal: $\lambda' - \lambda = \Delta\lambda \ll \lambda$.

(a) The m^{th}-order maxima for wavelengths λ and λ' occur at angles θ_m and θ'_m, respectively. What is the angular separation $\delta_m = \theta'_m - \theta_m$ of the m^{th}-order maxima for the two wavelengths? Simplify your answer based on the assumption that δ_m is a small angle. You may find it helpful to use a trig identity for $\sin(\alpha + \beta)$. Express your answer in terms of $m, \lambda, \Delta\lambda$, and d *only*.

(b) The *angular half-width* of a principal maximum is the angular separation between the principal maximum and the closest minimum. Find the angular half-width ϵ_m of the m^{th} order principal maximum for wavelength λ. Express your answer in terms of m, λ, N, and d *only*. [*Hint*: sum the finite geometric series $1 + e^{2\pi i/N} + e^{4\pi i/N} + \cdots + e^{2(N-1)\pi i/N}$ and then figure out that helps you find the first minimum past a principal maximum.]

(c) The grating can only resolve (separate) two nearly equal wavelengths in m^{th} order if the angular separation δ_m is sufficiently large. Explain why.

(d) A common but somewhat arbitrary criterion is that the wavelengths can be resolved if $\delta_m \geq \epsilon_m$. Using that criterion, what is the minimal value of $\Delta\lambda$ for which the waves can be resolved in m^{th} order? Express your answer in terms of m, λ, N, and d (as needed).

4 Reflection Grating

A *reflection* grating consists of an array of N equally spaced, identical narrow reflecting grooves or ridges on a non-reflecting surface. Light (wavelength $= \lambda$) from a distant source falls *normally* (that is, perpendicularly to the surface) on the grating. A distant observer observes light reflected at angle θ, see Fig. 2.

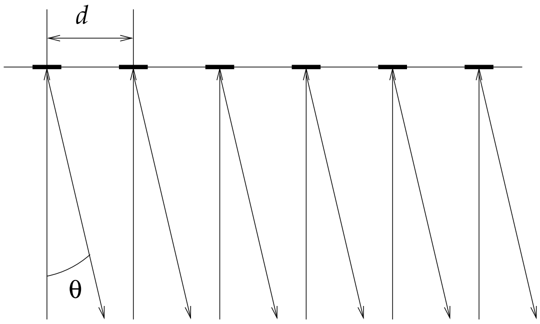

Figure 2: Reflection grating.

(a) What values of the angle θ_m correspond to principal maxima?

(b) Does the answer to (a) depend on N? Explain. If not, then what is the reason we generally use gratings (whether transmission gratings or reflection gratings) with very large N?

(c) Based on how the angles for principal maxima depend on λ, explain why the surface of a compact disc looks "rainbow-colored" when seen from an angle. [*Hint:* Natural light is a mixture of waves with wavelength spanning the whole spectrum from violet to red.]

(d) Suppose that the reflection grating is now illuminated with light from a distant source that is *off-axis*. That is, the incident light makes an angle $\phi \neq 0$ with the normal to the grating. At what angles θ_m do the principal maxima occur now?

(e) In the case of grazing incidence, as in Lab Experiment 3, ϕ is close to $\pi/2$. It is then more convenient to work with angles with respect to the grating *surface* rather than angles with respect to the normal. Let $\phi = \frac{\pi}{2} - \alpha$ and $\theta = \frac{\pi}{2} - \beta$. Find the angles β_m at which the principal maxima are found for a given value of α. Simplify your answer using small-angle approximations for α and β.

(f) Suppose that $d \gg \lambda$, as in Experiment 3. Use your answer to (e) to explain why we did the lab experiment using grazing incidence. [*Hint*: look at how the angular separation between principal maxima, $\beta_{m+1} - \beta_m$, depends on α.]

5 Checking some interesting limiting cases

(a) Let $I_a(\theta)$ be the expression for the intensity on the screen for a single slit of width a. Take the limit of $I_a(\theta)$ as $a \to 0$. Explain and interpret your result.

(b) Let $I_{N\times a}(\theta)$ be the expression for the intensity on the screen for N slits of width a and separation d. Simplify the form of $I_{N\times a}(\theta)$ in the special case $d = a$. Explain and interpret your result.