

**Complex numbers**

$$i^2 = -1$$

$$e^{ix} = \cos(x) + i \sin(x)$$

$$\underline{A} = A_r + iA_i = |\underline{A}|e^{i\phi}$$

$$|\underline{A}|^2 = A_r^2 + A_i^2 = \underline{A}^* \underline{A}$$

$$\tan \phi = A_i/A_r \quad \cos \phi = A_r/|\underline{A}|$$

$$|\underline{A} \cdot \underline{B}| = |\underline{A}| \cdot |\underline{B}|$$

$$\phi_{\underline{A} \cdot \underline{B}} = \phi_{\underline{A}} + \phi_{\underline{B}}$$

$$|\underline{A}/\underline{B}| = |\underline{A}|/|\underline{B}|$$

$$\phi_{\underline{A}/\underline{B}} = \phi_{\underline{A}} - \phi_{\underline{B}}$$

**Basic wave relationships**

$$f = 1/T \qquad \kappa = 1/\lambda$$

$$\omega = 2\pi f \qquad k = 2\pi\kappa$$

$$\omega = 2\pi/T \qquad k = 2\pi/\lambda$$

$$v = \lambda f$$

$$v = \omega/k$$

## Wave physics

	String	Sound
Degrees of freedom	$y(x,t)$	$s(x,t)$
Law of motion	$\frac{\partial T_y}{\partial x} = \frac{\partial}{\partial t} \left( \mu \frac{\partial y(x,t)}{\partial t} \right)$	$-\frac{\partial P}{\partial x} = \frac{\partial}{\partial t} \left( \rho_0 \frac{\partial s(x,t)}{\partial t} \right)$
Constitutive relations	$T_y = \tau \frac{\partial y}{\partial x}$	$P = P_0 - B \frac{\partial s}{\partial x}$
Momentum density	$p_y = \mu \frac{\partial y}{\partial t}$	$p_x = \rho_0 \frac{\partial s}{\partial t}$
Wave equation	$\tau \frac{\partial^2 y}{\partial x^2} = \mu \frac{\partial^2 y}{\partial t^2}$	$B \frac{\partial^2 s}{\partial x^2} = \rho_0 \frac{\partial^2 s}{\partial t^2}$
Wave speed	$c = \sqrt{\frac{\tau}{\mu}}$	$c = \sqrt{\frac{B}{\rho_0}}$
Bound. Cond. I	$y(x = x_0, t) = 0$ (fixed)	$s(x = x_0, t) = 0$ (closed)
Bound. Cond. II	$\frac{\partial y(x = x_0, t)}{\partial x} = 0$ (free)	$\frac{\partial s(x = x_0, t)}{\partial x} = 0$ (open)

## Solutions to Wave Equation

Standing Waves:  $y(x, t) = A_0 \cos(kx + \phi_1) \cos(\omega t + \phi_0)$