P214 Formula Sheet: Prelim III

Complex numbers

$$e^{ix} = \cos(x) + i\sin(x)$$
$$|\underline{A}|^2 = A_r^2 + A_i^2 = \underline{A}^*\underline{A}$$
$$|\underline{A}\cdot\underline{B}| = |\underline{A}|\cdot|\underline{B}|$$
$$|\underline{A}/\underline{B}| = |\underline{A}|/|\underline{B}|$$

Basic wave relationships

$$f = 1/T \qquad \kappa = 1/\lambda$$
$$\omega = 2\pi f \qquad k = 2\pi\kappa$$
$$\omega = 2\pi/T \qquad k = 2\pi/\lambda$$
$$c = \lambda f \qquad c = \omega/k$$

Wave physics

Quantity	String	Sound	E&M
Dynamical law(s)	$F_y = -\tau\frac{\partial y}{\partial x}$	$P = P_o - B\frac{\partial s}{\partial x}$	$\begin{cases} \frac{\partial}{\partial x}\left(\frac{1}{\mu}B_y\right) = \frac{\partial}{\partial t}\left(\epsilon_0 E_z\right) \\ \left(\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}\right) \end{cases}$
Wave equation	$\tau\frac{\partial^2 y}{\partial x^2} = \mu\frac{\partial^2 y}{\partial t^2}$	$B\frac{\partial^2 s}{\partial x^2} = \rho\frac{\partial^2 s}{\partial t^2}$	$\frac{1}{\mu_0}\frac{\partial^2 E_y}{\partial x^2} = \epsilon_0\frac{\partial^2 E_y}{\partial t^2}$
k.e.	$\frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2$	$\frac{1}{2}\rho\left(\frac{\partial s}{\partial t}\right)^2$	$\frac{1}{2}\epsilon_0 E^2$
p.e.	$\frac{1}{2}\tau\left(\frac{\partial y}{\partial x}\right)^2$	$\frac{1}{2}B\left(\frac{\partial s}{\partial x}\right)^2$	$\frac{1}{2\mu_0}B^2$
Intensity/Power	$-\tau\frac{\partial y}{\partial x}\frac{\partial y}{\partial t}$	$-B\frac{\partial s}{\partial x}\frac{\partial s}{\partial t}$	$\frac{1}{\mu_0}\vec{E}\times\vec{B}$

Wave equation and solutions

$$c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

$$\mp c \frac{\partial y}{\partial x} = \frac{\partial y}{\partial t}$$

$$y(x,t) = f(x - ct) + g(x + ct)$$

$$\left.\begin{array}{l} y_1(x<0,t) = f(x - v_1 t) + \frac{Z_1 - Z_2}{Z_1 + Z_2} f\left(-(x + v_1 t)\right) \\ y_2(x>0,t) = \frac{2Z_1}{Z_1 + Z_2} f\left(\frac{v_1}{v_2}(x - v_2 t)\right) \end{array}\right\} \text{(refl/trans)}$$

Quantum—Classical Correspondence

Quantum Quantity	Relationship	Newtonian Quantity
Wavelength	$\lambda = h/p$ $\hbar k = p$	Momentum
Frequency	$\nu = E/h$ $\hbar\omega = E$	Energy

Heisenberg Uncertainty Principle

$$\Delta x \, \Delta p \geq \hbar/2$$

Schrödinger equation and solutions

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$\psi(x) = Ae^{ik(x-a)} + Be^{-ik(x-a)},\ k = \sqrt{\tfrac{2m}{\hbar^2}(E - V)} \qquad \text{(classically allowed)}$$

$$\psi(x) = A'e^{-\alpha(x-a)} + B'e^{\alpha(x-a)},\ \alpha = \sqrt{\tfrac{2m}{\hbar^2}(V - E)}, \qquad \text{(classically forbidden)}$$

Geometric Series

$$\underline{a} + \underline{ax} + \underline{ax}^2 + \underline{ax}^3 + \ldots = \frac{\underline{a}}{1 - \underline{x}}$$

Interference

Multiple Narrow Slit Interference

Intensity: $I(\theta) = I_0 \frac{\sin^2(Nk\Delta R/2)}{\sin^2(k\Delta R/2)}$ where $\Delta R = d\sin\theta$, $N = \#$ of slits

Minima: $\Delta\Phi/(2\pi) = \frac{d}{\lambda}\sin\theta = \frac{n}{N}$ where $n = \pm1, \pm2, \ldots, \pm(N-1), \pm(N+1), \ldots$

Principal Maxima: $\Delta\Phi/(2\pi) = \frac{d}{\lambda}\sin\theta = n$ where $n = 0, \pm1, \pm2, \ldots$

Lessor maxima: $\Delta\Phi/(2\pi) = \frac{d}{\lambda}\sin\theta = \pm\frac{n+1/2}{N}$ where $n = 1, 2, \ldots, N-2, N+1, N+2, \ldots$

Finite Slit Diffraction

Intensity: $I(\theta) = I_o \frac{\sin^2(k\Delta r/2)}{(k\Delta r/2)^2}$ where $\Delta r = a\sin\theta$

Minima: $\Delta\phi/(2\pi) = \frac{a}{\lambda}\sin\theta = m$ where $m = \pm1, \pm2, \ldots$

Maxima: $\theta = 0$ **or** $\Delta\phi/(2\pi) = \frac{a}{\lambda}\sin\theta = \pm(m+1/2)$ where $m = 1, 2, \ldots, N-2, N+1, \ldots$

Multiple Finite Slit Diffraction

Intensity: $I(\theta) = \left(I_o \frac{\sin^2(k\Delta r/2)}{(k\Delta r/2)^2}\right)\left(\frac{\sin^2(Nk\Delta R/2)}{\sin^2(k\Delta R/2)}\right)$

Physical Constants

Quantity	Symbol	Value
Atomic mass unit	amu	1.66×10^{-27} **kg**
Electronic Mass	m_e	1×10^{-30} **kg**
Electronic Charge	e	$1.6 \times 10^{-19}\,\mathbf{C} = 1.6 \times 10^{-19}\,\mathbf{J/V}$
Permitivity of Free Space	ϵ_0	$8.9 \times 10^{-12}\,\mathbf{C^2/(N \cdot m^2)}$
Permeability of Free Space	μ_0	$4\pi \times 10^{-7}\,\mathbf{Ns^2/C^2}$
Planck's Constant	h	6.6×10^{-34} **J s**
	$\hbar$	1×10^{-34} **J s**
Speed of Light	c	3×10^8 **m/s**
Electron Volt	eV	1.6×10^{-19} **J**
Angstrom	Å	10^{-10} **m**